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Natural frequencies of functionally graded plates with porosities via a simple four variable plate theory: An analytical approach

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ABSTRACT

In this paper, the free vibration analysis of rectangular plates composed of functionally graded materials with porosities is investigated based on a simple first-order shear deformation plate theory. The network of pores in assumed to be empty or filled by low pressure air and the material properties of the plate varies through the thickness. Using Hamilton's principle and utilizing the variational method, the governing equations of motion of FG plates with porosities are derived. Considering two boundary layer functions, the governing equations of the system are rewritten and decoupled. Finally, two decoupled equations are solved analytically for Lévy-type boundary conditions so as to obtain the eigenfrequencies of the plate. The effects of porosity parameter, power law index, thickness-side ratio, aspect ratio, porosity distribution and boundary conditions on natural frequencies of the plate are investigated in detail.

1. Introduction

A lot of investigations have been performed to study the mechanical characteristics of functionally graded (FG) beams and plates owing to their interesting features such as lack of delamination and cracking. Abrate [1] showed that the behavior of functionally graded plates can be determined by interfering the results which have been compiled for homogeneous materials meaning the natural frequency, buckling load and static deflection of FG plates are proportional to those of homogeneous isotropic plates. Employing a two-dimensional higher-order theory, Matsunaga [2] obtained the distribution of modal stresses and modal displacements in thickness direction by satisfying the surface boundary conditions of a FG plate. Dozio [3] proposed an analytical approach based on Carrera unified formulation for free vibration analysis of Lévy-type plates made of FG materials in which the effective material properties are estimated according to Mori-Tanaka scheme and rule-of-mixture. Free vibration analysis of four types of thick FG skew and rectangular plates has been performed by Zhao et al. [4] who used mesh-free kernel particle functions to approximate 2D displacement fields. Using the method of separation of variables, Hosseini Hashemi et al. [5] solved the highly coupled partial differential equations dealing with the free vibration response of thick functionally graded plates with continuously varying properties through the plate thickness. Yang and Shen [6,7] obtained numerical solutions by using semi-analytical DQ and modal superposition method for free vibration and transient response of FG thin plates with initial in-plane actions subjected to distributed impulsive lateral loads without or under thermal environment. Neves et al. [8] employed a meshless technique based on collocation with radial basis functions to deal with the static and free vibration analysis of isotropic and sandwich functionally graded plates. Yang and Chen [9] developed an exact analytical formulation for FGM beams with edge cracks according to Euler-Bernoulli beam theory, and studied the natural frequencies, critical buckling loads, and the corresponding mode shapes of such beams.

A comprehensive assessment of shear deformation plate theories against free vibration, buckling, and static deflection of rectangular plates of uniform thickness is presented by Saidi and co-authors [10–15] who used some auxiliary functions in order to present an analytical method for computing the natural frequencies, critical buckling loads and static deflection of the plates. Benachour et al. [16] used a four variable refined plate theory for free vibration analysis of clamped/simply supported FG plates and considered the effects of aspect ratio, thickness-side ratio and varying gradient on eigenfrequencies of the plates. Thai and Choi [17] investigated the free vibration behavior of FG rectangular plates resting on elastic foundation by developing a refined shear deformation plate theory which contains only four unknowns. Yang and co-authors [18–20] presented Galerkin-based solutions for nonlinear and chaotic vibration analysis of functionally graded rectangular plates with/without cracks on the basis of Reddy's third-order shear deformation plate theory.

Compared with the numerical analysis of FG plates, the investigations concerned with the static and dynamic response of porous

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structures are limited in number. Among those research papers, Theodorakopoulos and Beskos [21] obtained the dynamic response of a simply supported rectangular plate made of porous materials to harmonic lateral load and considered the effects of porosity, inertia and permeability on the system's response. On the basis of Biot-Allard theory, Etchessahar et al. [22] solved the governing equations of motion which deal with the bending vibration of air saturated poroelastic plates by using Galerkin method. Magnucka-Blandzi [23,24] studied the static and dynamic behavior of porous-cellular circular plates having macroscopic properties varying across the plate thickness based on a non-linear displacement field. Jabbari and co-authors [25–28] used various types of plate theories to analyze the buckling of porous circular plates, where the material properties vary through the thickness according to a cosine rule. Chen and his co-authors investigated the elastic buckling and static bending [29], nonlinear free vibration [30], nonlinear vibration and postbuckling [31] performances of functionally graded porous beams based on Timoshenko beam theory and Ritz method. They used a direct iterative algorithm in order to extract the nonlinear frequencies, critical buckling loads, postbuckling equilibrium paths, transverse deflections of the beams. Wattanasakulpong and Ungbhakorn [32] provided a numerical solution by means of differential transformation method (DTM) for linear and nonlinear vibration of elastically restrained ends beams composed of functionally graded materials with porosities. Saidi and co-authors [33–39] presented a series of analytical solutions to study the free vibration and buckling behaviors of porous plates by using the various plate models. The buckling of functionally graded piezoelectric plates with porosities resting on elastic foundation is studied by Barati et al. [40] who used a refined four variable plate theory and some admissible functions and examined the effects of external voltage, power-law index, porosity distribution, and elastic foundation parameters on the system's response. Ebrahimi and Jafari [41] proposed a four variable shear deformation beam theory to investigate the thermo-mechanical vibrations of simply supported temperature dependent FG beams with porosities. To the best of the authors' knowledge, free vibration analysis of FG plates with porosities using a simple four variable refined plate theory and an analytical procedure is presented by no researchers yet.

In view of the above, the aim of the present paper is to develop a genuine mathematical model for free vibration analysis of Lévy-type porous FG rectangular plates within the framework of a simple first-order shear deformation plate theory. The formulation is based on the four variable refined plate theory of Thai and Choi [17] and the solution procedure is developed with the help of two auxiliary functions. Following this approach, the governing equations of the system are expressed in terms of these two new functions and converted into two decoupled partial differential equations. The obtained results are compared with those presented by other reference papers and are found to agree well. Extensive parametric studies for different boundary conditions are presented. It is believed that the reported results can probably be helpful for other researchers in order to compare their findings related to similar problems.

2. Functionally graded materials with porosities

Consider a rectangular plate made of FG materials which contains porosities in its structure. The Cartesian coordinate system is established so that $0 \le x_1 \le l_1$, $0 \le x_1 \le l_2$ and $0 \le x_3 \le h$ as depicted in Fig. 1(a). Taking into account the power law for volume fraction, the modified effective mechanical properties of a FGM-I can be written as [32]

$$P = P_b + (P_t - P_b)\left(\frac{2x_3 + h}{2h}\right)^n - \frac{e}{2}(P_t + P_b) \tag{1}$$

Here, P denotes the effective material property. e and n are porosity parameter and power law index, respectively. When $e = n \equiv 0$, the material degenerates into an isotropic one. Also, the subscripts b and t refer to the bottom and top surface of the plate, respectively. It is to be noted that the above relation describe a medium in which the pores are evenly distributed. The second scenario for porosity distribution is given by [32]

$$P = P_b + (P_t - P_b)\left(\frac{2x_3 + h}{2h}\right)^n - \frac{e}{2}(P_t + P_b)(1 - \frac{2|x_3|}{h}) \tag{2}$$

In this type of material, which will be referred as FGM-II, porosity phases spreading frequently nearby the middle zone of the cross-section and the amount of porosity seems to be linearly decrease to zero at the top and bottom of the cross-section [41]. Based on the principle of multi-step sequential infiltration technique that can be used to produce FGM samples, the porosities mostly occur at the middle zone. At this zone, it is difficult to infiltrate the materials completely, while at the top and bottom zones, the process of material infiltration can be performed easier and leaves less porosity [32].

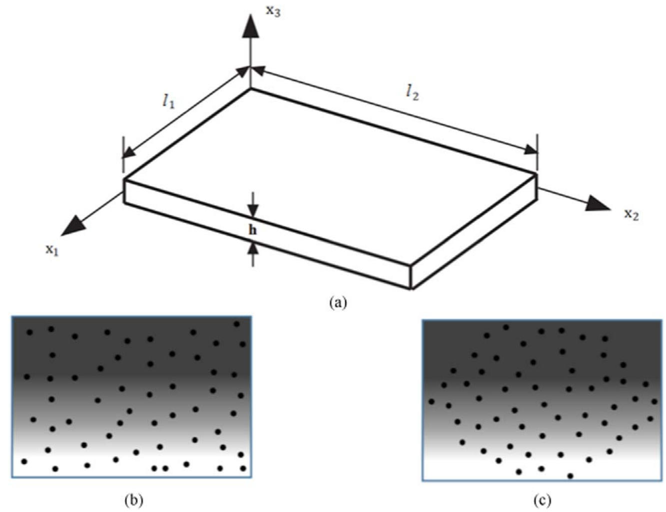

Fig. 1. (a) Geometry of a porous FG plate (b) FGM-I cross section [32] (c) FGM-II cross section [32].

3. Dynamic equations of the plate

In order to analyze the natural frequency response of the plate, a simple-first order shear deformation plate theory is assumed as follow [17]

$$\begin{aligned} U(x_1, x_2, x_3, t) &= u_0(x_1, x_2, t) - x_3 w_{b,1} \\ V(x_1, x_2, x_3, t) &= v_0(x_1, x_2, t) - x_3 w_{b,2} \\ W(x_1, x_2, x_3, t) &= w_b(x_1, x_2, t) + w_s(x_1, x_2, t) \end{aligned} \tag{3}$$

Here, u_0 and v_0 are the in-plane displacements of mid-plane in x_1 and x_2 direction, respectively. The transverse displacement of the plate is divided into bending and shears parts. According to the infinitesimal strain-displacement relations, the strains associated with the above displacement field are

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{11}^0 \\ \varepsilon_{22}^0 \\ \gamma_{12}^0 \end{Bmatrix} + x_3 \begin{Bmatrix} \kappa_{11}^b \\ \kappa_{22}^b \\ \kappa_{12}^b \end{Bmatrix}$$
$$\begin{Bmatrix} \gamma_{13} \\ \gamma_{23} \end{Bmatrix} = \begin{Bmatrix} \gamma_{13}^0 \\ \gamma_{23}^0 \end{Bmatrix} \tag{4}$$

where