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A refined finite element method for bending of smart functionally graded plates

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ABSTRACT

This study presents a finite element (FE) formulation based on four-variable refined plate theory for a bending analysis of functionally graded (FG) plates integrated with a piezoelectric fiber reinforced composite (PFRC) actuator under electrical and mechanical loadings. The four-variable refined plate theory accounts for a parabolic variation of the transverse shear stresses across the plate thickness, which satisfies zero traction conditions on the plate free surfaces. The principle of minimum potential energy is used to derive the weak form of governing equations, and a 4-node nonconforming rectangular plate element with eight degrees of freedom (DoFs) per node is introduced for discretizing the domain of the bending variables. Some benchmark problems are also solved using the developed MATLAB code. A comparison of the results of the obtained displacements and stresses with the exact and other numerical solutions shows good agreement, thereby proving the simplicity and efficiency of the present finite element (FE) solutions. In addition, the effects of several parameters on the results, including the thickness ratio, Young's modulus ratio, the types of boundary conditions, and the distribution and amount of loadings, are investigated.

1. Introduction

In recent years, functionally graded materials (FGMs), as a new class of composites, have been developed. They have gained considerable attention among other composites owing to their superior features. The material properties of FGMs show a smooth and continuous change from one surface to another in one or more directions, eliminating the stress concentration found in laminated composites. This property is achieved in plate and shell structures usually by gradually changing the composition of the constituent materials through the thickness. The idea behind the use of FGMs was first proposed by material scientists in the Sendai area of Japan [1]. FGMs are now widely used in many structural applications, such as aerospace, nuclear, mechanical, civil, biomedical, electrical, chemical, and automotive fields. Because of their widespread applications, FG plates have received significant attention, and a variety of plate theories such as classical plate theory (CPT), first-order shear deformation theories (FSDTs), and higher-order shear deformation theories (HSDTs) have been employed for modeling these structures.

CPT is the simplest plate theory, and does not take into account the shear deformation effects. As such, it only gives acceptable results for a static analysis of thin plates. Owing to the deficiencies of CPT, FSDTs have been introduced, in which the transverse shear stress is assumed to be constant through the plate thickness. These theories cannot satisfy the zero traction conditions on free surfaces, and also require a shear correction factor in their formulations. The accuracy of these theories depends on the determination of a proper value for the shear correction factor. To overcome the limitations of FSDTs, various HSDTs have been proposed, such as the trigonometric shear deformation theory, the hyperbolic shear deformation theory, and the two-variable refined plate theory. The two-variable plate theory is simple and efficient. It contains parabolic transverse shear stresses across the plate thickness, satisfying zero traction conditions on the free surfaces of the plate. This theory was first presented by Shimpi [2] for a bending analysis of isotropic plates, and was then extended to orthotropic and laminated plates by Shimpi and Patel [3], Thai and Kim [4], and Kim et al. [5]. Free vibration and buckling analyses of plates were conducted by Shimpi and Patel [6] and Kim et al. [7], respectively. Mechab et al. [8] employed this theory for a bending analysis of FG plates. Rouzegar and Abdoli presented finite element formulations based on a two-variable refined plate theory for bending [9], free vibration [10], and buckling [11] analyses of isotropic and orthotropic plates. In the two-variable refined plate theory, the middle surface of the plate is assumed to be unstrained, and only the bending effects are considered.

In the four-variable refined plate theory, two other parameters associated with the in-plane displacements of the plate middle surface are

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added. A static analysis of FG plates using the four-variable refined plate theory was presented by Mechab et al. [12] and Thai and Choi [13].

Many investigations have been focused on the analysis of different responses of plate structures bonded with piezoelectric layers owing to the superior piezoelectric properties such as a quick response, large power generation, operational ability at very low temperatures, and vacuum capability. One of the first investigations on piezoelectric plates was conducted by Tiersten and Mindlin [14]. The governing equations for a piezoelectric layer were introduced by Tiersten [15]. Several piezoelectric materials such as PZT and PVDF are available; however, these materials have certain limitations such as low piezoelectric constants, shape control, and high specific acoustic impedance. Because of these shortcomings, a new material called a piezoelectric fiber-reinforced composite (PFRC) has been introduced [16]. Mallik and Ray [17] applied the notion of unidirectional piezoelectric fiber reinforced composite materials and presented their effective properties.

Ray and Sachade [18] conducted a static analysis of FG plates with a piezoelectric layer using the finite element method (FEM) and first-order shear deformation theory. Ray and Sachade [19] presented an exact solution for the problem of an FG plate with a layer of piezoelectric fiber-reinforced composite. Panda and Ray [20] presented a nonlinear finite element analysis of FG plates integrated with patches of PFRC. Shiyekar and Kant [21] developed an electromechanical higher-order analytical model for a flexure analysis of FG plates integrated with PFRC layers. Behjat *et al.* [22] conducted static and dynamic analyses of FG piezoelectric plates under mechanical and electrical loadings using FEM and FSDT. Rouzegar and Abad presented an analytical solution for the flexure of composite plates [23] and the free vibration of FG plates [24] integrated with piezoelectric layers using the four-variable refined plate theory.

In this study, a finite element formulation based on the four-variable refined plate theory was developed for a static analysis of FG plates integrated with a layer of a PFRC actuator. The present plate theory is an efficient HSDT, and has a simple formulation in comparison to common shear deformation plate theories. In this theory, a parabolic variation of the transverse shear strain and stress across the plate thickness is considered, and zero traction conditions are satisfied on the plate surfaces. Therefore, there is no need for a shear correction factor in the formulation. The piezoelectric layer acts as an actuator, and the plate is subjected to electrical and mechanical loadings. A 4-node nonconforming rectangular plate element with eight degrees of freedom (DoFs) per node is introduced for discretizing the domain of the bending variables. The presented approach was validated by solving of some benchmark problems and comparing the results with the exact and other FE solutions. The effects of different parameters such as the thickness ratio, Young's modulus ratio, various types of boundary conditions, and the distribution and amount of loadings are also examined.

2. Formulation

Fig. 1 shows a rectangular smart plate of length a and width b, which contains an FG substrate and a layer of PFRC attached at the top surface of the substrate. The thickness of the FG substrate is h and the thickness of piezoelectric layer is h_p, which is small compared with h. The origin of the right-handed Cartesian coordinate system (x, y, z) is located at the corner of the middle plane of the FG substrate. The hybrid plate is subjected to both electrical and mechanical loadings.

2.1. Displacement and strain

According to the four-variable refined plate theory, the displacement components u, v, and w in the x, y, and z directions are introduced below [23]:

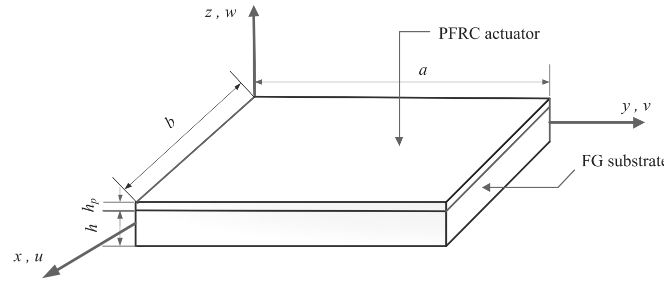

Fig. 1. Geometry of the rectangular FG substrate attached with a PFRC actuator at the top.

$$u(x,y,z) = u_0(x,y) - z\frac{\partial w_b}{\partial x} + z\left[\frac{1}{4} - \frac{5}{3}\left(\frac{z}{h}\right)^2\right]\frac{\partial w_s}{\partial x},$$
$$v(x,y,z) = v_0(x,y) - z\frac{\partial w_b}{\partial y} + z\left[\frac{1}{4} - \frac{5}{3}\left(\frac{z}{h}\right)^2\right]\frac{\partial w_s}{\partial y},$$
$$w(x,y) = w_b(x,y) + w_s(x,y), \tag{1}$$

where u_0 and v_0 are the in-plane displacements of the mid-plane in the x and y directions, and w_b and w_s are the bending and shear components of the transverse displacements, respectively. The strain-displacement relations are given as

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_{xy}\end{Bmatrix} = \begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix} + z\begin{Bmatrix}\chi_x^b\\ \chi_y^b\\ \chi_{xy}^b\end{Bmatrix} + f\begin{Bmatrix}\chi_x^s\\ \chi_y^s\\ \chi_{xy}^s\end{Bmatrix},\quad \begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} = g\begin{Bmatrix}\gamma_{yz}^s\\ \gamma_{xz}^s\end{Bmatrix},\quad \varepsilon_z = 0, \tag{2}$$

where ε_x and ε_y are normal in-plane strains along the x and y axes, ε_{xy} is the in-plane shear strain, and γ_{xz} and γ_{yz} are the transverse shear strains, respectively, i.e.,

$$\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix} = \begin{Bmatrix}\dfrac{\partial u_0}{\partial x}\\ \dfrac{\partial v_0}{\partial y}\\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x}\end{Bmatrix},\quad \begin{Bmatrix}\chi_x^b\\ \chi_y^b\\ \chi_{xy}^b\end{Bmatrix} = \begin{Bmatrix}-\dfrac{\partial^2 w_b}{\partial x^2}\\ -\dfrac{\partial^2 w_b}{\partial y^2}\\ -2\dfrac{\partial^2 w_b}{\partial x\partial y}\end{Bmatrix},$$
$$\begin{Bmatrix}\chi_x^s\\ \chi_y^s\\ \chi_{xy}^s\end{Bmatrix} = \begin{Bmatrix}-\dfrac{\partial^2 w_s}{\partial x^2}\\ -\dfrac{\partial^2 w_s}{\partial y^2}\\ -2\dfrac{\partial^2 w_s}{\partial x\partial y}\end{Bmatrix},\quad \begin{Bmatrix}\gamma_{yz}^s\\ \gamma_{xz}^s\end{Bmatrix} = \begin{Bmatrix}\dfrac{\partial w_s}{\partial y}\\ \dfrac{\partial w_s}{\partial x}\end{Bmatrix},$$
$$f = -\frac{1}{4}z + \frac{5}{3}z\left(\frac{z}{h_t}\right)^2,\quad g = \frac{5}{4} - 5\left(\frac{z}{h_t}\right)^2, \tag{3}$$

where h_t is the total thickness of the substrate and piezoelectric layer.

2.2. Constitutive equation

The FG material considered in this study is isotropic at any point, and the Young's modulus changes exponentially through the thickness according to the following exponential relation [25]:

$$E = E_0\, e^{\lambda(z+\frac{h}{2})}, \tag{4}$$

where E_0 is the Young's modulus of the bottom surface of the FG plate, and λ is a parameter describing the inhomogeneity of the FG plate, i.e.,