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Free vibration of joined conical-conical shells

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ABSTRACT

Free vibration analysis of a joined shell system composed of two conical shells is analysed in this research. It is assumed that the system of joined shell is made from a linearly elastic isotropic homogeneous material. Both shells are unified in thickness. To capture the through-the-thickness shear deformations and rotary inertias, first order theory of shells is accompanied with the Donnell type of kinematic assumptions to establish the general motion equations and the associated boundary and continuity conditions with the aid of Hamilton's principle. The resulted system of equations are discreted using the semi-analytical generalised differential quadrature (GDQ) method. Considering various types of boundary conditions for the shell ends and intersection continuity conditions, an eigenvalue problem is established to examine the vibration frequencies as well as the associated mode shapes. After proving the efficiency and validity of the present method for the case of thin isotropic homogeneous joined shells, some parametric studies are carried out for the system of combined moderately thick conical-conical.

1. Introduction

A system of joined conical-conical shell has increasing application in submarine, civil and aerospace engineering. It is well accepted that, close to the joined section, localised and severe bending moments are produced when the shell is subjected to sudden loadings. Vibrations induced by such loadings may results in fatigue phenomenon. Therefore, it is of high interest and importance to understand the vibration characteristics of joined shells to establish the fundamental requirements for a safe design.

A large number of publications deal with the vibration analysis of elementary class of shells, e.g. conical, cylindrical or spherical shell elements. On such topics, therefore, wealth books are documented [1–3]. On the other hand, in comparison to the elementary shells, researches on vibration analysis of joined shells are a few but not so many. Primary research on the free vibration analysis of joined shells belongs to Hu and Raney [4]. Analytical results of this study are accompanied with the experimental justification. In this research, a joined cantilever cylindrical-conical shell is analysed. This piece of work reveals the existences of a V-shaped notch at the intersection of the joined shell which is the source of stress concentration or fatigue phenomenon. Lashkari and Weingarten [5] proposed a finite element procedure accompanied with an experimental technique to analyse the free vibration of a shell-cone structure. It is shown that the different geometrical portions of the segmented shell can vibrate independently of each other, i.e., it is possible for one portion to vibrate without exciting the other. Chang and Greif [6] proposed a Fourier series based solution for free vibration analysis of segmented cylindrical shells. Results of this study are presented for cantilever and simply supported shell systems. It is assumed that system contains two shells of unified mean radius and different thicknesses. Irie et al. [7] presented a strategy to analyse the vibration of a joined cylindrical-conical shell element. The transfer matrix of the shell is expressed conveniently by the power series and the frequency equations are derived for a given set of boundary conditions at the edges. As an special case, free vibration characteristics of an annular plate-cylindrical shell system is also analysed. A substructure synthesis method based on the state space approach is established by Tavakoli and Singh [8] to analyse the free vibration of joined/Hermetic shells. Various combinations of shell structures, such as cylinder-cylinder, cone-cylinder, hemisphere-hemisphere and hermetic capsules are modelled. In this research, Páde approximation is used for matrix exponentiation of the governing equations associated with each shell substructure. Based on a higher order semi-analytical finite elements method, free vibration characteristics of combined cylindrical-conical shell systems is investigated by Sivadas and Ganesan [9].

Based on the first order shell theory, Patel et al. [10] analysed a three segmented shell, i.e. a cone-cylinder-cone shell in free vibration regime. Shell is made of composite laminated materials and a semi-

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analytical finite element method is developed for the establishment of eigenvalue problem. Lee et al. [11] performed an investigation on the free vibration of combined cylindrical-spherical shells. Various case of boundary conditions are considered in this research. Rayleigh- and Ritz-based solutions are developed to establish the eigenvalue problems associated with the natural frequencies and mode shapes of a thin Flügge shell system. It is shown that the vibrational behavior of the joined spherical-cylindrical shell structure is independent of the shallowness of a hemispherical shell whereas the length of the cylindrical shell is effective in the vibrational behavior of the joined hemispherical-cylindrical shell. Efraim and Eisenberger [12] analysed the free vibration and mode sequence of segmented thin Reissner-Naghdi shells using the dynamic stiffness matrix of an axisymmetric shell. Based on the two different solution methods, Caresta and Kessissoglou [13] obtained the free vibration characteristics of joined conical-cylindrical shell system. Both thin class of Donnell-Mushtari and Flügge shells are analysed. Solution method of this research is based on the polynomial expansion of displacement field with respect to the axial direction of the shell. Natural frequencies are presented for some selective cases of boundary conditions. Kang [14] analysed the free vibration of joined cylindrical-conical shells within the framework of three dimensional elasticity theory. In this research, thickness variation of the shell is also included. Multi-term Ritz solution is used to extract the eigenvalue problem form the associated potential and kinetic energies. Employing a variational approach, Qu et al. [15] analysed the free vibration of joined cylindrical-conical shell system with classical or non-classical boundary conditions. The thin shell assumptions of Reissner-Naghdi theory are used as the fundamental theoretical assumptions. The interface continuity and geometric boundary conditions are approximately enforced by means of a modified variational principle and least-squares weighted residual method. Qu and his co-authors also applied their previous method [15] to the free vibration analysis of ring-stiffened joined conical-cylindrical shell systems [16], joined conical-cylindrical-spherical shell systems [17], joined cylindrical-spherical shell with elastic-support boundary conditions [18] and spherical-cylindrical-spherical shells [19]. Kouchakzadeh and Shakouri [20] developed the method of Caresta and Kessissoglou [13] for vibration analysis of cross-ply laminated conical-conical shell systems. Shakouri and Kouchakzadeh [21] also analysed the free vibration of thin conical-conical shell system using the power-series solution system. Their research also includes experimental investigation on the free vibration characteristics of joined conical-conical shells with both ends free. Ma et al. [22] investigated the free and force vibration response of a system of joined conical-cylindrical shell using a Fourier-Ritz formulation. The solution method of this research is general and may be used for arbitrary combinations of boundary conditions. In the mentioned work each of the displacement components are expanded invariantly as a modified Fourier series, which is composed of a standard Fourier series and closed form supplementary functions introduced to accelerate the convergence of the series expansion and remove all the relevant discontinuities at the boundaries and the junction between the two shell components. Chen et al. [23] analysed the free and forced vibration of ring stiffened joined conical-cylindrical shell with arbitrary boundary conditions. Kinematic of the shell are estimated according to the Flügge shell theory. Furthermore, instead of adopting the smeared out method and treating the ring stiffeners as beams, the stiffeners with rectangular cross section are treated as discrete members and the equations of motion of annular plate are used to describe the motion of stiffeners. Sarkheil and Saadat Foumani [24] discussed the effects of rotation on the free vibration characteristics of joint conical-cylindrical shell structures. Solution method of this research is also based on the power series solution technique. Sarkheil et al. [25] obtained the free vibration response of a system containing n conical shells using the classical shell theory formulation. Similar to the previous investigation of the authors, the solution method of this research is also based on the power series method. Recently, using the thin shell model assumptions and employing the matrix transformation technique and power series solution method, Sarkheil et al [26] performed an investigation to obtain the frequencies of a system of rotating joined shell containing arbitrary number of conical shells. Xie et al. [27] obtained the frequencies of thin stepped conical shell system using the method of power series solution.

The above literature survey reveals that most of the investigations on the subject of joined shell systems belong to the combined cylindrical-conical shell system. There are only a few works dealing with the free vibration response of joined conical-conical shells which are limited to thin class of shells. Present study aims to investigate the free vibration characteristics of a joined conical-conical shell structure using a shear deformable shell model suitable for moderately thick shells. Due to the special configuration of the structure, free vibration of single annular plate, single conical or cylindrical shell, joined annular plate/cylindrical shell, joined annular plate/conical shell, joined conical/cylindrical shell and joined conical/conical shell systems may be analysed as especial cases. First order shell theory accompanied with the Donnell type of kinematic assumptions are used to establish the motion equations and the associated boundary conditions. A semi-analytical procedure based on Fourier expansion along the circumferential direction and GDQ discretization along the meridian direction is developed to discrete the motion equations. GDQ method is also applied to the intersection continuity and boundary conditions. A system of homogeneous eigenvalue problems is established which may be useful to examine the frequencies and mode shapes of a joined conical-conical shells. After validating the proposed solution method via some comparison studies, a series of parametric studies are performed to examine the influences of cone angles, cone thickness, cone radii and boundary conditions.

2. Governing equations

Consider a joined circular conical-conical shell made of an isotropic homogeneous material of uniform thickness h, end radii R_1, R_3, intersection radius R_2, slanted lengths L_1 and L_2 and vertex half angles α_1 and α_2. Meridional, circumferential and normal directions of each conical shell are denoted by $0 \le x^i \le L_i$, $i = 1, 2$, $0 \le \theta \le 2\pi$ and $-h/2 \le z \le +h/2$, respectively. The adopted coordinates system (x^i, θ, z), geometric characteristics and sign convention of the joined shell are depicted in Fig. 1.

To capture the through the thickness shear deformations and rotary inertia effects of the conical shell, the first order shear deformation theory (FSDT) of shells is used to formulate the governing equations of the shell. Based to FSDT, components of the displacement on a generic point may be represented according to the mid-surface characteristics such that

$$\begin{aligned} u^i(x^i, \theta, z, t) &= u_0^i(x^i, \theta, t) + z\varphi_x^i(x^i, \theta, t) \\ v^i(x^i, \theta, z, t) &= v_0^i(x^i, \theta, t) + z\varphi_\theta^i(x^i, \theta, t) \\ w^i(x^i, \theta, z, t) &= w_0^i(x^i, \theta, t) \end{aligned} \tag{1}$$

In the above equation u, v and w are the meridional, circumferential and through-the-thickness displacements, respectively. A subscript 0 indicates the characteristics of the mid-surface. Besides φ_θ and φ_x are, respectively, the transverse normal rotations about the x and θ axes. Furthermore, herein and all the rest, superscript i takes the values 1 and 2 and is associated to the ith shell segment.

According to FSDT the components of strain field on an arbitrary point of the conical shell may be obtained in terms of those belong to the mid-surface of the shell and change of curvatures. Consequently, one may write [28]