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On the nonlinear dynamics of shell structures: Combining a mixed finite element formulation and a robust integration scheme

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ABSTRACT

In this work, we present an approach to analyze the nonlinear dynamics of shell structures, which combines a mixed finite element formulation and a robust integration scheme. The structure is spatially discretized with extensible-director-based solid-degenerate shells. The semi-discrete equations are temporally discretized with a momentum-preserving, energy-preserving/decaying method, which allows to mitigate the effects due to unresolved high-frequency content. Additionally, kinematic constraints are employed to render structural junctions. Finally, the method, which can be used to analyze blades of wind turbines or wings of airplanes effectively, is tested and its capabilities are illustrated by means of examples.

1. Introduction

The complexity of current and new shell structures in combination with the increased computation capacity encouraged the development and application of fully nonlinear shell formulations. In this context, time-domain analysis involving large displacements, large rotations and large strains due to dynamic loads plays a major role. The discrete equations of shells are in fact very stiff and therefore, the calculation of long-term response could be very problematic, even for well-established commercial codes. Achieving robustness requires the development of new methods that must annihilate the unresolved high-frequency content, warranting evenly the preservation of the underlying physics. Certainly, the accomplishment of these features is very challenging.

Dvorkin and Bathe [1] developed a four-node shell element for general nonlinear analysis, which is applicable to the analysis of thin and thick shells. Bathe and Dvorkin [2] discussed the requirements for linear and nonlinear analysis. Büchter and Ramm [3] addressed the controversy between solid-degenerate approaches and shell theories. Bütcher et al. [4] enabled the introduction of unmodified three-dimensional constitutive laws by means of the enhanced assumed strain method proposed by Simo and Rifai [5]. Simo and Tarnow [6,7] developed a time integration scheme for the dynamics of elastic solids and shells, which preserves, independently of the time step size, the linear momentum, the angular momentum and the total energy. Choi and Paik [8] presented the development of a four-node shell element for the analysis of structures undergoing large deformations. Betsch and Stein [9,10] developed a four-node shell element that incorporates unmodified three-dimensional constitutive models. This element was improved by means of the enhanced assumed strain method proposed by Simo and Armero [11]. Bischoff and Ramm [12] formulated a geometrically nonlinear version of the enhanced assumed strain approach in terms of Green-Lagrange strains. Sansour et al. [13] combined a geometric exact shell theory and an integration scheme that preserves the linear momentum, the angular momentum and the total energy. Kuhl and Ramm [14] developed a generalization of the energy-momentum method developed within the framework of the generalized α method. Armero and Romero [15,16] developed a family of schemes for nonlinear three-dimensional elastodynamics that exhibits controllable numerical dissipation in the high-frequency range. For a fixed and finite time step, the method produces a correct picture of the phase space even in the presence of dissipation. Sansour et al. [17] dealt with a dynamic formulation of shells and the development of a robust energy-momentum integration scheme. Romero and Armero [18] extended the previously introduced method for the dynamics of geometrically exact shells. Proofs of the numerical properties in the full nonlinear range were also provided. Bauchau et al. [19] developed energy-preserving/decaying schemes for the simulation of multibody systems including shell components. Vu-Quoc and Tan [20] developed an eight-node solid-degenerate shell element and tested it with several integration methods. Sansour et al. [21] modified an existent method to deal with material nonlinearities. Ozkul [22] presented a finite element for dynamic analysis of shells of general shape. Ziemčík [23] presented a four-node shell element to analyze lightweight smart structures. Leyendecker et al. [24] extended a framework for the computational treatment of rigid bodies and nonlinear beams to the realm of nonlinear shells. Vaziri [25] studied the response of shell structures under large

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deformations and presented a review of the current state-of-the-art with practical suggestions. Carrera et al. [26] considered the mixed interpolation of tensorial components, which was extended to model shells with variable kinematics. Wu [27] described the nonlinear dynamic behavior of shell structures by means of a vector form intrinsic formulation. Ahmed and Sluys [28] presented a three-dimensional shell element for the dynamic analysis of laminated composites. Pietraszkiewicz and Konopińska [29] reviewed different theoretical, numerical, and experimental approaches to model, analyze and design compound shell structures with junctions. Reinoso and Blázquez [30] developed an eight-node solid-degenerate shell element, which was reformulated in the context of composite structures. Recently, Caliri et al. [31] presented a very detailed literature review on plate and shell theories for composite structures with highlights to the finite element method.

In this work, we present an approach to analyze the nonlinear dynamics of shell structures, which relies on the combination of a mixed finite element formulation and a robust integration scheme that is presented in a differential-algebraic setting. The structure is spatially discretized with extensible-director-based solid-degenerate shells. The shear locking and the artificial thickness strains are cured by means of the assumed natural strain method, but the enhancement of the strain field in the thickness direction and the cure of the membrane locking are achieved by means of the enhanced assumed strain method. This director-based approach allows to consider unmodified three-dimensional constitutive laws by only improving the director field. Due to the adopted kinematic description, the treatment of folded shells is very simple and the combination with rigid bodies and beams is straightforward (this is valid for rotationless frameworks). Similar approaches for eight-node solid-degenerate shells would require to reparametrize the director field in terms of the upper and lower displacement fields, which would be surely effective, but certainly more elaborate. However, objective and comprehensive comparisons between the four-node extensible-director-based solid-degenerate shell and the eight-node displacement-based solid degenerate shell are not part of this work. The resulting semi-discrete equations are temporally discretized by means of a momentum-preserving, energy-preserving/decaying method, which allows to mitigate the undesirable effect due to unresolved high-frequency content providing robustness without destroying the precision of the solution. Finally, some interesting constraints to render more complicated structures by means of junctions are introduced, and its null-space treatment is briefly described. In few words, from a methodological point of view, the novelty of this work is the combination of a mixed finite element for shells, the time integration with a momentum-preserving, energy-preserving/decaying method and the null-space projection method into a single common unifying framework, which may be used very effectively to analyze blades of wind turbines or wings of airplanes. To our best knowledge, there is not a single work, in which all these three topics are combined with a similar setting in the context of shell structures.

The remaining is organized as follows: Section 2 presents the adopted mechanical framework, comprising a general description, the spatial discretization that relies on a mixed formulation, the temporal discretization that relies on a robust scheme, the treatment of various interesting constraints and the presentation of the discrete equations. In Section 3, we present five examples taken from literature, which were chosen to show the potentialities and capabilities of the exposed ideas. Finally, concluding remarks and future work are given in the Section 4.

2. Mechanical framework

In this section, we present the necessary mathematical tools to deal, from a purely mechanical point of view, with shell structures. First, we roughly outline the fundamentals that are needed to establish a starting point. Second, we introduce the shell kinematics and the spatial discretization based on a mixed finite element formulation. Third, we introduce the temporal discretization starting from the energy-momentum-conserving algorithm and the modifications needed to build a momentum-preserving, energy-preserving/decaying algorithm. Fourth, we provide a brief exposition of some interesting constraints, which are used to render more complicated structures. And lastly, we present the governing equations for the fully discretized problem in their final implementation form.

2.1. Generalities

Let us assume a continuum body characterized by a chosen reference set denoted by $\mathscr{B}_0$, this is an open set of $\mathbb{R}^3$, whose configuration and velocity are described at time t by the vectors $\boldsymbol{x}(t) \in \mathscr{X}_t \subseteq \mathbb{R}^3$ and $\boldsymbol{v}(t) \in \mathscr{V}_t \subseteq \mathbb{R}^3$, respectively. In addition, let us assume that the body is subjected to a finite-dimensional set of constraints $\boldsymbol{h}(\boldsymbol{x}) = \boldsymbol{0} \in \mathbb{R}^n$, with $n \in \mathbb{N}$, that only accounts for integrable restrictions. The dynamic behavior of the flexible system within the bounded time interval $[t_1, t_2] \subset \mathbb{R}_0^+$ can be formulated with the Hamilton principle as

$$\overline{\delta\mathfrak{S}} = \int_{t_1}^{t_2} \int_{\mathscr{B}_0} [\langle \delta\boldsymbol{v}, \boldsymbol{l}(\boldsymbol{x}) - \boldsymbol{l}(\boldsymbol{v})\rangle - \langle \delta\boldsymbol{x}, \boldsymbol{f}^{\text{int}}(\boldsymbol{x}, \boldsymbol{S}^\sharp) + \dot{\boldsymbol{l}}(\boldsymbol{v}) - \boldsymbol{f}^{\text{ext}}(\boldsymbol{x}) + \boldsymbol{H}(\boldsymbol{x})^T\boldsymbol{\lambda}\rangle - \langle \delta\widetilde{\boldsymbol{E}}_\flat, \boldsymbol{S}^\sharp\rangle - \langle \delta\boldsymbol{\lambda}, \boldsymbol{h}(\boldsymbol{x})\rangle]\mathrm{d}\mathscr{B}\,\mathrm{d}t = 0\ , \tag{1}$$

where $\overline{\delta\mathfrak{S}}$ is merely the infinitesimal increment of the action functional, which may be not an actual variation due to the presence of non-conservative external fields, and $\langle\cdot,\cdot\rangle$ is an appropriate dual pairing. $\delta\boldsymbol{x} \in T_{\boldsymbol{x}(t)}\mathscr{X}_t$ and $\delta\boldsymbol{v} \in T_{\boldsymbol{v}(t)}\mathscr{V}_t$ are admissible variations of the configuration and velocity vectors, respectively. The displacement-based momentum density $\boldsymbol{l}(\boldsymbol{x})$ and the velocity-based momentum density $\boldsymbol{l}(\boldsymbol{v})$ map elements of $T_{\boldsymbol{v}(t)}\mathscr{V}_t$ to elements of $\mathbb{R}$. The time rate of the velocity-based momentum density $\dot{\boldsymbol{l}}(\boldsymbol{v})$, the internal force density $\boldsymbol{f}^{\text{int}}(\boldsymbol{x}, \boldsymbol{S}^\sharp)$ and the external force density $\boldsymbol{f}^{\text{ext}}(\boldsymbol{x})$ map elements of $T_{\boldsymbol{x}(t)}\mathscr{X}_t$ to elements of $\mathbb{R}$. $\boldsymbol{H}$: $\mathbb{R}^n \times T_{\boldsymbol{x}(t)}\mathscr{X}_t \to \mathbb{R}$ is the Jacobean matrix of $\boldsymbol{h}(\boldsymbol{x})$, $\boldsymbol{\lambda}$: $[t_1, t_2] \to \mathbb{R}^n$ belongs to the space of curves with no boundary conditions, the well-known Lagrange's multipliers, and $\delta\boldsymbol{\lambda}$ represents an admissible variation of the multipliers. The strain $\overline{\boldsymbol{E}}_\flat \in \overline{\mathscr{E}} := \{\overline{\boldsymbol{E}}_\flat \in T^*_{\boldsymbol{x}(\boldsymbol{0})}\mathscr{X}_0 \times T^*_{\boldsymbol{x}(\boldsymbol{0})}\mathscr{X}_0 | 2\overline{\boldsymbol{E}}_\flat = [\boldsymbol{\phi}_t \circ (\boldsymbol{\phi}_0)^{-1}]^*\boldsymbol{G}[\boldsymbol{x}(t)] - \boldsymbol{G}[\boldsymbol{x}(0)]\}$ is the displacement-based part of the Green-Lagrange strain tensor, in which $\boldsymbol{G}[\boldsymbol{x}(0)]$ is the metric tensor in the original configuration, $\boldsymbol{G}[\boldsymbol{x}(t)]$ is the metric tensor in the current configuration and $[\boldsymbol{\phi}_t \circ (\boldsymbol{\phi}_0)^{-1}]^*(\cdot)$ denotes the pullback from the current configuration to the original one by means of the regular motion $\boldsymbol{\phi}_t \circ (\boldsymbol{\phi}_0)^{-1}$. The strain $\widetilde{\boldsymbol{E}}_\flat \in \widetilde{\mathscr{E}} := \{\widetilde{\boldsymbol{E}}_\flat \in T^*_{\boldsymbol{x}(\boldsymbol{0})}\mathscr{X}_0 \times T^*_{\boldsymbol{x}(\boldsymbol{0})}\mathscr{X}_0 \big| \text{skew}(\widetilde{\boldsymbol{E}}_\flat) = \boldsymbol{0}, \int_{\mathscr{B}_0} \langle \widetilde{\boldsymbol{E}}_\flat, \boldsymbol{S}^\sharp\rangle d\mathscr{B} = 0\}$ is an enhancement and $\delta\widetilde{\boldsymbol{E}}_\flat \in T_{\widetilde{\boldsymbol{E}}_\flat(t)}\widetilde{\mathscr{E}}_t$ represents an admissible variation, assuming that the strain space possesses a manifold structure. From the linear combination of both parts, *i.e.* $\overline{\boldsymbol{E}}_\flat + \widetilde{\boldsymbol{E}}_\flat$, results the Green-Lagrange strain tensor $\boldsymbol{E}_\flat$, which is energetically conjugated by an appropriate stress definition $\boldsymbol{S}^\sharp(t) \in \mathscr{S} := \{\boldsymbol{S}^\sharp \in T_{\boldsymbol{x}(\boldsymbol{0})}\mathscr{X}_0 \times T_{\boldsymbol{x}(\boldsymbol{0})}\mathscr{X}_0 | \boldsymbol{S}^\sharp = \partial_{\boldsymbol{E}_\flat}\Psi(\boldsymbol{E}_\flat)\}$, the second Piola-Kirchhoff stress tensor, and $\Psi(\boldsymbol{E}_\flat)$ is the material law. The symbol $\flat(\sharp)$ indicates that a second-rank tensor is doubly covariant (contravariant).

2.2. Shell kinematics and spatial discretization

The position at time t of any given point belonging to the shell can be written as

$$\boldsymbol{x}(\boldsymbol{\theta}; t) = \overline{\boldsymbol{x}}(\theta^1, \theta^2; t) + \theta^3\frac{\vartheta}{2}\boldsymbol{d}(\theta^1, \theta^2; t), \tag{2}$$

in which $\overline{\boldsymbol{x}} \in \mathbb{R}^3$ is the position vector of the middle surface, ϑ represents the thickness of the shell and $\boldsymbol{d}$ is an extensible director, which admits multiplicative decomposition, *i.e.* $\boldsymbol{d} = d\,\widehat{\boldsymbol{d}}$ with $d \in \mathbb{R}^+$ and $\widehat{\boldsymbol{d}} \in S^2$. $\boldsymbol{\theta} = (\theta^1, \theta^2, \theta^3)$ is a set of parameters chosen in the way that $\overline{\boldsymbol{\theta}} = (\theta^1, \theta^2, 0)$ describes the middle surface and $\boldsymbol{x} = \boldsymbol{x}(\boldsymbol{\theta}; t)$ is the given parameterization rule in time and space, see Fig. 1. For instance, we can choose $\boldsymbol{\theta}$ to span the domain $\square$ such as

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