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Generalisation of the Ayrton-Perry formula for the global-distortional-local buckling of thin-walled members

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ABSTRACT

The paper introduces a generalisation of the Ayrton-Perry formula for thin-walled members, covering uncoupled and coupled instabilities of global, distortional and local type. This semi-analytical solution is based on the Generalised Beam Theory (GBT) and it provides the maximum carrying capacity in elastic domain for simply-supported thin-walled members with initial geometric imperfections, under various loading conditions. Direct generalised Ayrton-Perry formulae are developed for the buckling described by one or a combination of two pure imperfection/deformation modes; for more than two coupled modes, a simple incremental procedure is provided.

1. Introduction

Despite its many simplifying assumptions, the Ayrton-Perry formula (APF) is a very popular method to find the buckling resistance of steel members, due to its clear mechanical background, simplicity and flexibility. Therefore it has been introduced in many modern design standards as Eurocode 3 Chapter 1-1 (EC3) [1] for basic cases as the flexural buckling of columns, and the lateral-torsional buckling of beam and beams-columns, all of them belonging to the global buckling type. The flexural buckling of columns is the only case with a strong theoretical and experimental background [2–6]. Regarding the lateral-torsional buckling, the design curves are based mainly on numerical simulations [7], since the experimental testing is difficult to perform (few results have been reported [8]) and the analytical description is still under investigation [9,10]. Even if the classical APF works in elastic domain and only for global buckling modes, successful attempts have been made to adapt it for the plastic–elastic interactive buckling introducing the local failure modes into the elastic global behaviour of the member [11,12].

The goal of the research presented in this paper is to create a Generalised APF (GAPF) working in elastic domain, for all three buckling types: global, distortional and local. The main instrument used by the author is the Generalised Beam Theory (GBT). Originally created by Schardt [13,14] and extensively developed by Camotim, Silvestre et al. [15–17], GBT is a specialized method capable to perform a wide variety of structural analyses, the most important being the study of stability behaviour of thin-walled members. GBT has the ability to decompose the member buckling deformation into a linear combination of pure deformation modes, which accounts, at cross-section level, for both rigid-body (global) motions and in-plane (distortional and local) deformations. A very important feature of GBT is the unified formulation for all deformation modes. A conventional GBT analysis involves two steps: (i) a cross-section analysis leading to the GBT cross-section deformation modes (the *pure* modes) and the corresponding modal mechanical properties and (ii) a member stability analysis to obtain the critical loading factors and the associated mode shapes (the amplitude functions). Only the first step is used in this paper, and next, the GBT cross-section deformation modes together with the corresponding modal mechanical properties are introduced in GAPF. Until now, promising results were obtained for the buckling of simply-supported thin-walled members, under axial compression although the theoretical developments cover any type of constant loading along the bar. The member deformation and also the initial geometric imperfections are represented by a single (or a combination of) GBT deformation modes of global, distortional and local type, and they are introduced by using pure sinusoidal amplitude functions of the GBT cross-section deformation modes with arbitrary number of halfwaves along the member. Since these pure sinusoidal functions satisfy only the simple-support boundary condition, the proposed procedure is for the time being limited to simply supported members. For local buckling usually characterised by a large number of halfwaves along the member, the influence of different boundary conditions is generally not significant. For distortional buckling characterised by a much lower number of halfwaves, the influence could be significant. As for the global buckling, the classical APF handles different boundary conditions by application of the well-known effective length factor. General end boundary

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conditions can be represented by specially selected longitudinal amplitude functions, as the ones previously used in connection with the Finite Strip Method ([18,19]), and this expansion is currently under work.

No initial stresses are considered. For the time being GAPF covers one or a combination of two deformation modes; for more than two coupled modes, a simple numerical incremental procedure is provided.

It is important to underline that the present formulation provides the carrying capacity of the thin-walled members only in *elastic* domain, therefore it does not provide the maximum carrying capacity (the ultimate load) of the member as the classical APF does in correlation with the Effective Width Method. In other words the formulation does not explore the post-buckling strength reserve which is significant for both distortional and local buckling. The analysis will stop at first yield, which often is reached by the flexural component of normal longitudinal stresses in local buckling, thus ignoring the strength reserve related with the membrane stress components. The formulation is also limited to small deformations and linear distribution of the membrane normal longitudinal stress. Extending the formulation for nonlinear stress distribution due to local post-buckling is currently under work. A significant advantage of the proposed semi-analytical formulation over FEA, besides its speed, is the capability of providing the modal participation in elastic domain for each pure buckling mode considering initial geometric imperfection analysis.

2. The original Ayrton-Perry formula – short review

The APF was originally developed for the flexural buckling of elastic prismatic columns uniformly compressed, having one halfwave sinusoidal imperfection shape. The load carrying capacity is given by the start of yielding at the most compressed fibre [2]. The problem presented in Fig. 1 is described by the following equilibrium differential equation:

$$EIv'' + N(v + v_0) = 0 \tag{1}$$

where EI is the appropriate lateral stiffness, $v(x)$ and $v_0(x)$ are the lateral displacement and imperfection functions, respectively, N is the compressive force and $(\,)' = d(\,)/dx$.

Notice that by double derivation, the equation can be written:

$$EIv^{IV} + N(v'' + v_0'') = 0 \tag{2}$$

The functions $v(x)$ and $v_0(x)$ are introduced as follows:

$$v(x) = a\sin\left(\frac{\pi x}{L}\right) \quad v_0(x) = a_0\sin\left(\frac{\pi x}{L}\right) \quad v_{tot} = v + v_0 = a_{tot}\sin\left(\frac{\pi x}{L}\right) \tag{3}$$

where L is the bar's length, and a, a_0, a_{tot} are the amplitudes of the lateral displacement, the initial imperfection and the total lateral deformation of the longitudinal axis on y direction, respectively. The total amplitude is found as:

$$a_{tot} = \frac{1}{1 - N/N_{cr}}a_0 \tag{4}$$

At midspan, the most compressed fiber reaches the yield stress:

$$\frac{N}{A} + \frac{N\cdot a_{tot}}{W} = f_y \tag{5}$$

where A is the cross-section area, W is the elastic sectional modulus and f_y is the yield stress. Introducing the notations $\sigma_b = N/A$ for the compressive stress from axial loading, Eq. (5) becomes:

$$\sigma_b + \sigma_b\frac{N_{cr}}{N_{cr} - N}\frac{a_0 A}{W} = f_y \tag{6}$$

Introducing also the generalised imperfection factor $\eta = a_0A/W$ and the elastic critical compressive stress $\sigma_{cr} = N_{cr}/A$, the original APF is obtained:

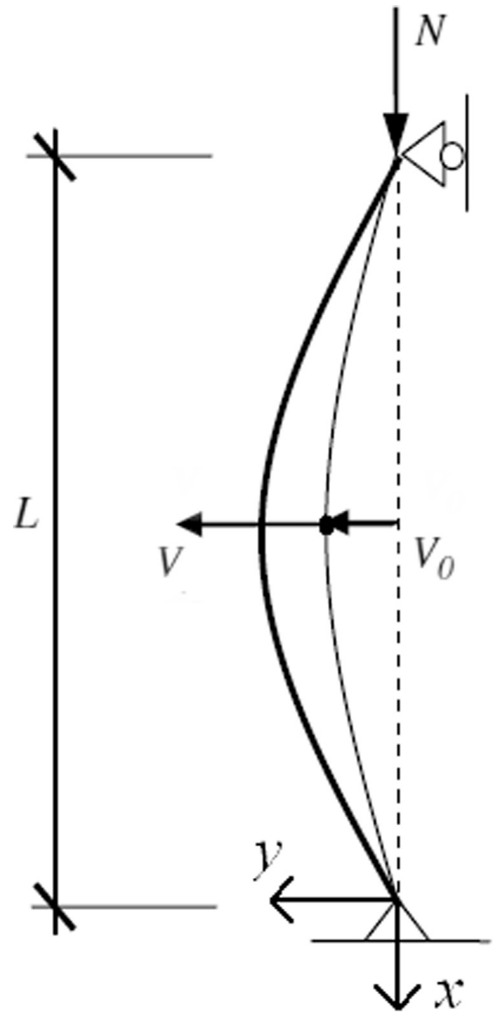

Fig. 1. The model of the original APF [9].

$$(\sigma_{cr} - \sigma_b)(f_y - \sigma_b) = \sigma_b\sigma_{cr}\eta \tag{7}$$

Using the EC3 notations [1], the APF can be rewritten as follows:

$$\chi^2 + \chi\left(-1 - \frac{1}{\lambda^2} - \frac{\eta}{\lambda^2}\right) + \frac{1}{\lambda^2} = 0 \tag{8}$$

where $\lambda = \sqrt{f_y/\sigma_{cr}}$ is the member slenderness, and $\chi = \sigma_b/f_y$ is the buckling reduction factor. The solution yields the design buckling curves, where the generalised imperfection factor was defined by extensive investigations involving experimental, numerical and probabilistic analyses [6].

3. The Generalised Beam Theory–short review

The local coordinate system x-s-z and the corresponding displacement field u-v-w are shown in Fig. 2 for an arbitrary thin-walled member.

In classical GBT, one uses the Kirchhoff-Love plate theory, and the membrane shearing strains together with the transversal extensions are neglected [13]. According to GBT, any displacement is considered as a linear combination of m pure deformation modes of global-distortional-local type, each one expressed as a product of two functions: