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Nonlinear vibration analysis of pre-twisted functionally graded microbeams in thermal environment

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ABSTRACT

The large amplitude free vibration behavior of pre-twisted functionally graded (FG) microbeams in thermal environment is investigated based on the modified strain gradient theory (MSGT) in conjunction with the first-order shear deformation theory (FSDT). The geometrical nonlinearity effects are taken into account in the sense of von Kármán nonlinear kinematic assumptions. The Chebyshev–Ritz method combined with harmonic balance method is employed to derive the nonlinear algebraic eigenfrequency equations of the microbeams subjected to different boundary conditions. The material properties are assumed to be temperature-dependent and graded in the thickness direction. After numerically demonstrating the fast rate of convergence and accuracy of the method, the effects of different geometrical and material parameters on the nonlinear free vibration behavior of pre-twisted FG microbeams are investigated. It is found that the effect of twist angle on the hardening or softening of the microbeams depend on the boundary conditions, and the largest and smallest values of nonlinear to linear frequency ratios belong to simply supported and fully clamped microbeams, respectively.

1. Introduction

Among the different classes of composite materials, the mechanical behavior of functionally graded materials (FGMs) has become a topic of interest to researchers due to their unique characteristic properties. Some of the impressive properties of FGMs, resulting from the smooth and continuous variation of properties along certain dimensions, improve stress distributions, results higher fracture toughness and enhanced thermal resistance [1,2]. So, FGMs are suitable materials which can be used in a wide range of applications such as micro/nano-electro-mechanical systems in thermal environment.

One of the structural elements widely used in microengines, microturbomachinery and micromachining [3–8], the ultrasonic piezo-electronic motor [9], and the development of a micromotor for in vivo medical procedures [10], is the pre-twisted microbeams. In the above mentioned applications, the pre-twisted microbeams usually experience vibratory motion with relatively large amplitude in thermal environment. In order to protect these structural elements from the thermal erosion and also to enhance their natural frequencies, it is preferred to make them from the FGMs. Therefore, the study of the linear and nonlinear vibratory behavior of pre-twisted FG microbeams in thermal environment is necessary to achieve functional guideline for the accurate design of the above devices.

It is well established that the classical continuum mechanics theories can not capture the size effects and consequently are not able to accurately predict the mechanical behavior of the micro/nanoscale structures. On the other hand, the atomistic simulations need much computational efforts and are limited to simple systems and in particular numerical solutions can be found only for systems with a few numbers of atoms and molecules. To overcome these drawbacks, different higher-order theories containing additional material constants such as Eringen's nonlocal elasticity theory [11,12], the couple stress theory (CST) [13] and the strain gradient theory (SGT) [14,15] have been introduced to incorporate the size effect into classical continuum mechanic theory [16]. In order to reduce the number of additional material constants, Yang et al. [17] and Lam et al. [18] modified the couple stress theory and the strain gradient theory, respectively.

Among the different nonlocal continuum theories, the modified couple stress theory (MCST) and modified strain gradient theory (MSGT) have been usually used to analyze the micro-sized structural elements; see for example Refs. [19–45]. However, Shu and Fleck [45] showed that the couple stress theory, which is a general form of the MCST, underestimates the size effect because it only employs the rotation gradient and neglects the other gradients. In spite of the strain gradient theory, which needs five additional length scale, the MSGT contains only three additional length scale parameters corresponding to

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the microstructure rotation gradient, the microstructure dilation gradient, and the microstructure stretch gradient for isotropic linear elastic materials [18]. In addition, it can be shown that the MCST can be recovered as a special case of the MSGT by setting to zero two of the three material length scale parameters of the MSGT [18]. In the past years, some researcher investigated the linear free vibrations of the FG microbeams based on the MSGT; see for example Refs. [20,22,23]. On the other hand, the nonlinear vibration of beams in thermal environment is a practically and theoretically important problem and the researches in this field are continued [46–51]. For the brevity purpose, in the following, some of the current research works regarding the nonlinear vibrations of microbeams are briefly reviewed.

Vatankhah et al. [25] employed the MSGT and considered the geometrical nonlinearities as well as the nonlinear external forces such as van-der-Waals force to investigate the size-dependent nonlinear forced vibration of Euler–Bernoulli microbeams. They solved the nonlinear governing equations analytically by utilizing the perturbation techniques. Ghayesh et al. [26] studied the nonlinear forced vibrations of an Euler-Bernoulli microbeam by applying the MSGT. The Galerkin method together with the pseudo-arc length continuation technique were employed to solve the resulting nonlinear governing equations. In another work, Ghayesh et al. [27] investigated the geometrically nonlinear size-dependent behavior of a Timoshenko microbeam based on the MCST by employing the same numerical technique. Wang et al. [28] presented the geometrically nonlinear free vibration analysis of the Euler–Bernoulli microbeams using the MCST. They applied the Kantorovich and shooting methods to solve the nonlinear governing equations.

Şimşek [29] developed a non-classical beam theory for the static and nonlinear vibration analysis of microbeams supported on a three-layered nonlinear elastic foundation within the framework of the MCST and Euler–Bernoulli beam theory. He obtained approximate analytical expressions for the nonlinear frequency of the microbeams with pinned–pinned and clamped–clamped end conditions by means of He's variational method. Using the MSGT, Setoodeh and Afrahim [30] incorporated the size effect into nonlinear Euler–Bernoulli beam theory and studied the size-dependent nonlinear vibration behavior of FG micro-pipes conveying fluid. They obtained the nonlinear fundamental frequency by applying the Galerkin method and the homotopy analysis method. Sahmani et al. [31] conducted a numerical analysis to predict the size-dependent geometrically nonlinear free vibration characteristics of third-order shear deformable FG microbeams based on the MSGT. The nonlinear governing differential equations of motion subjected to different end boundary conditions were discretized by employing generalized differential quadrature method and the resulting nonlinear algebraic eigenvalue equations were solved using a direct iterative process.

Şimşek [32] studied the geometrically nonlinear free vibration of axially functionally graded (AFG) Euler–Bernoulli microbeams with immovable ends based the MCST. He developed an approximate closed form solution for the nonlinear governing partial differential equations subjected to pinned–pinned and clamped–clamped boundary conditions by utilizing the Galerkin method and He's variational method. Dai et al. [33] simulated the nonlinear forced vibration of cantilever Euler-Bernoulli microbeams based on the MCST. They used the Galerkin method to solve the governing equation and showed how the existence of an intermediate linear spring support increases the resonance frequency and decrease the resonance amplitudes. Hu et al. [34] developed a nonlinear theoretical model for cantilevered micropipes/microbeams conveying fluid to explore the possible size-dependent nonlinear responses based on the MCST together with Euler-Bernoulli beam theory. The geometric nonlinearities, the gravity, and the effect of loose supports at the downstream end were considered in the derived governing partial differential equation, which were discretized using the Galerkin method.

Shafiei and his co-worker studied the geometrically nonlinear free vibration of non-uniform axially functionally graded (AFG) microbeams [35] and also, imperfect uniform and non-uniform functionally graded (FG) microbeams [36] based on the MCST and Euler–Bernoulli beam theory. They employed the generalized differential quadrature method and direct iterative method to obtain the nonlinear natural frequencies. Farokhi et al. [37] analyzed the nonlinear size-dependent dynamics of a microcantilever under base excitation using the MCST. They solved the nonlinear equation of motion by applying a weighted-residual method together with a continuation technique and a direct time-integration scheme. Setoodeh and Rezaei [38] studied the geometrically nonlinear free vibration of FG nano/micro beams based on the MCST together with both Euler-Bernoulli and Timoshenko beam theories. The nonlinear partial differential equations were solved semi-analytically using the Galerkin method and the method of homotopy analysis. Ansari et al. [39] developed a nonlinear microstructure-dependent third-order shear deformable beam model based on the most general form of Mindlin's strain gradient elasticity theory (SGT) and the von Kármán hypothesis to describe the nonlinear free and forced vibrations as well as nonlinear bending and postbuckling of the FG microbeams. The variational differential quadrature method together with the periodic time differential operators and the pseudo arc-length continuation method were utilized to solve the governing equations.

Şimşek [40] proposed a novel size-dependent beam model for nonlinear free vibration of the FG nanobeams with immovable ends based on the nonlocal strain gradient theory and Euler-Bernoulli beam theory in conjunction with the von-Kármán's geometric nonlinearity assumptions. By neglecting the axial inertia and using the Galerkin's method, the partial nonlinear differential equation was reduced to an ordinary nonlinear one with cubic nonlinearity and a closed-form solution for nonlinear frequency was extracted.

The linear free vibrations of the pre-twisted microbeams were studied by Ghorbani Shenas et al. [41] and Mohammadimehr [42], recently. Ghorbani Shenas et al. [41] employed the MSGT together with the FSDT of beams to investigate the vibrational characteristics of rotating pre-twisted cantilever microbeams in thermal environment. Mohammadimehr et al. [42] used the MSGT and the classical beam theory to model the vibration and wave propagation in the pre-twisted microbeams supported on Pasternak foundation. Also, in an important application, Mustapha and Zhong [21] employed the modified couple stress theory in conjunction with the Euler–Bernoulli beam theory to study the effects of the rate of twist angle and the material length scale on the bending wave propagation characteristics of the pre-twisted microscale beams. They presented the results for the spectrum curve, the cut-off frequency, the phase speed and the group velocity of a

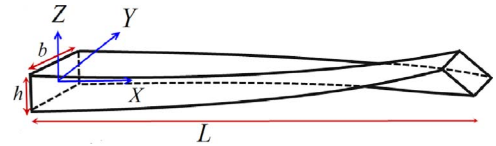

Fig. 1. The geometry and global coordinate system of the pre-twisted FG microbeams.

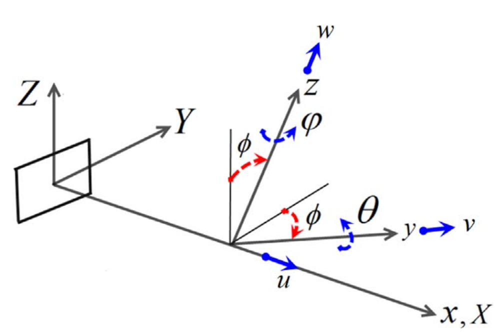

Fig. 2. The global and local coordinate system together with the rotation components of the pre-twisted microbeams.