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Geometric nonlinear analysis of prismatic shells using the semi-analytical finite strip method

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ABSTRACT

The present study sheds light on geometric nonlinear static analysis of prismatic shells using the semi-analytical finite strip method. A new computational model, which includes a fully nonlinear compound strip with a longitudinal and transverse stiffener, has been presented. Furthermore, strips with non-uniform characteristics in the longitudinal direction have been used in nonlinear analysis. This has, to the best knowledge of authors, never been reported. Also, this paper describes the design and implementation of eighteen ideal boundary conditions using three different longitudinal and six well-known transverse displacement interpolation functions.

The results of the presented study were obtained using an open-source software and multi-purpose software Abaqus. Moreover, the accuracy of the applied computational approach has been verified by comparison with results from the literature. An excellent agreement of displacement fields is achieved for large deflection analyses of plates with a hole and stiffeners as well as for shells with a stepped thickness in the longitudinal direction. Additionally, results from post-buckling analyses of thin-walled structures, a snap-through and snap-back behavior of shallow shells, are matched. The work presented here has profound implications for future studies of the finite strip deployment.

1. Introduction

Prismatic shells are architecturally impressive and increasing in popularity. They appear as load-bearing components in diverse engineering systems or in nature forms and represent a broad class of engineering structures such as plates, walls, orthotropic plates, cylindrical shells, folded plates, thin-walled girders, etc. They are structures with zero curvature in the longitudinal direction while a cross-section in the plane perpendicular to the longitudinal direction is polygonal or curved with a high slenderness ratio. As a consequence, a high efficiency in shells design can be obtained. However, those structures have a nearly chaotic and complex behavior which exceeds limits of linearity. Consequently, an advanced computational approach, including the material and geometric nonlinearities, is behooved. Therefore, it is of utmost importance to know a specific response of those sensitive structures. Although those nonlinearities generally occur simultaneously coupled, for certain classes of prismatic shells geometric nonlinear effects precede material ones, which justifies analysis of geometric nonlinearity separately.

Analysis of prismatic structures is a highly specialized discipline in modern structural engineering. The most general method for solving those structures is the finite element method (FEM), but for some specific geometries and loads, the finite strip method (FSM) proves more efficient. In this paper, we are mainly interested in the nonlinear static analysis using the FSM, which will be juxtaposed with the FEM due to its wide acceptance and versatility. A more detailed review of the FEM is beyond the scope of this paper but it is noted, however, that some of the basics are given in [1,2], while the authors [3,4] show recent trends in the FEM and its integration with the computer aided designs using the isogeometric approach.

The FSM was pioneered by Cheung [5]. The main idea is to approximate the displacement field in the longitudinal direction with a series of continuous functions and to discretize in the transverse direction, which leads to a set of finite strips connected at nodal lines.

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This approach classifies the method as semi-analytical. If free vibration eigenfunctions are used for the approximation, the equations of balance can be uncoupled for some cases due to their orthogonality. Theoretical foundations of the FSM are presented in [6–8]. It is already known that the FSM is recognized as an alternative to the FEM for regular geometries and applied in many areas of structural analyses and designs. Linear static and dynamic analyses, including free vibration and bifurcation problems, have been studied in detail by [9–11] and [12], producing a continuous and significant enhancement of the FSM. Buckling analyses are of particular interest here since it is the first step for an imperfection sensitive nonlinear analysis. Recent contributions are related to the development of the constrained FSM [13,14]. A significant effort is devoted to nonlinear analysis and the main contributions are briefly presented.

One of the first works in the nonlinear analysis using the FSM is [15] and the post-locally-buckled behavior of prismatic plate structures due to a uniform end compression (modeled as a uniform end shortening) has been analyzed. Due to the incompatibility of functions at corners of a structure, the approach is limited to local analyses. A non-uniform end compression is introduced in [16], while the first investigation of local and global buckling mode interactions of thin-walled beams is shown in [17]. Membrane displacements are decomposed into a linear (pre-buckling) and nonlinear (post-buckling) part. The main assumption is that only a simplified structure with the length equal to the half-wavelength of buckled mode of a beam should be analyzed. Later in this paper, we will verify this assumption by comparison with other numerical results.

Moreover, a local post-buckling behavior of plates has been successfully investigated using the semi-energy FSM [18], followed by a set of works where the method is used for analysis of many nonlinear problems of plates [19,20]. In this method, the classic interpolation function is inserted into the von Karman's compatibility equation, which is solved exactly. The similar approach is used for the development of an exact finite strip, which is introduced in [21] and implemented for a wide range of structures [22–24]. The method is named "exact" since an exact buckling mode is introduced into the von Karman's equations and it is accurate for structures for which one series term is sufficient to describe their behavior.

In addition, the FSM has been satisfyingly used to pass a limit point for the first time in [25], while the first paper that deals with an analysis of thick plates is [26]. An approach to an elastic-plastic nonlinear analysis of cold-formed sections is proposed in [27]. Although the compatibility of displacements at section corners is not fully satisfied, an appropriate choice of interpolation functions overcomes it. Analysis of a nonlinear response of a cable-stayed bridge by adopting eigenfunctions of an equivalent continuous beam for interpolation functions is proposed in [28]. Next, papers [29,30] deal with the nonlinear analysis of thin plates and thin-walled structures with emphasis on the necessity for inclusion of a full harmonic coupled formulation, which has been often neglected. It is stressed out that the coupled formulation must be applied for analyses where the buckling mode interaction is expected. An effort for improvement of this approach via parallelization is presented in [31,32]. Furthermore, the validation of nonlinear FSM analysis of C sections with the experimental results has been materialized in [33]. Local buckling of stiffened plates is considered in [34]. It is pointed out that the nonlinear equations of balance are coupled and it can be swimmingly avoided by neglecting the non-diagonal blocks of a tangent stiffness matrix.

Additionally, a significant improvement of nonlinear FSM has been accomplished in [35]. It has been proved that the classic longitudinal membrane interpolation functions are not well-suited for modeling of free ends. By the addition of a linear function to the sine series, an analysis of structures with an arbitrary end shortening has been empowered. Nonlinear analysis of layered composite plates is presented in [36] with the comparison of trigonometric and spline functions, while also a generally spaced spline has been introduced. It turns out that the semi-analytical FSM is more accurate for certain cases while a spline, especially a generally spaced, is more functional. Also, a post-buckling of composite plates under a combined compression and shear loading is examined in [37]. Nonlinear behavior of viscoelastic plates is analyzed in [38,39], where bubble functions have been used instead of classic polynomials.

The spline finite strip method (SFSM) is extensively developed and widely used. Its application to geometric nonlinear analyses of plates is presented in [40], while an extension to non-elastic buckling of beams, columns, and plates is given in [41]. The post-buckling behavior of cylindrical shells with imperfections has been considered in [42]. It is discussed that a displacement-dependent load is a controversial theme in the FEM and demonstrated that this type of load modeling is necessary for some problems. Furthermore, the same authors in [43] state that for the transverse membrane displacement component the third order polynomial is needed. In addition, geometric nonlinear analysis of thin-walled structures using the SFSM is presented in [44], where boundary conditions have been derived in detail. Also, papers [45,46] handle with static and dynamic geometric nonlinear analysis of stiffened plates using the SFSM. The presented procedure for modeling stiffeners is similar to the compound strip approach, which will be used in this research. In [47,48], an isoparametric nonlinear analysis of thin-walled structures using the SFSM is given. Adaptability of the method has been emphasized via a detailed analysis of structural perforations.

All the aforementioned facts show that the semi-analytical FSM, and FSM in general, is a powerful method for solving many sophisticated behaviors of structures in the linear and nonlinear analyses. However, the semi-analytical FSM still has many disadvantages and they are mostly due to its approximation of displacement field with a series of continuous functions. Most of the previous theoretical works were mainly focused on problems of initiation of internal supports, stiffeners, and non-uniform characteristics along the strip only in linear analyses. Notwithstanding the application of those elements in the linear analysis, their implication in a nonlinear region is a crucial factor for the improvement of the FSM. Besides, the boundary conditions other than the simply supported are rarely used in literature and their contributions and effects should be elaborated. In order to take the advantage over the other methods, the semi-analytical FSM has to be enhanced.

The main aim of this paper is to overcome those disadvantages using the harmonic coupled semi-analytical FSM in geometric nonlinear analyses. A new computational model has been presented and the finite strip technique has been extended to investigate structures of complex geometries and arbitrary boundary conditions. Here we would like to emphasize that the total Lagrangian approach, large displacement, finite strain (but small) and infinitesimal rotation theory have been used. Also, some of the preliminary results are given in [49–53], but a significant generalization and improvement are presented in this paper. Moreover, the presented approach is implemented in an open-source software framework. The accuracy of the applied mathematical model and computational approach has been verified by direct comparisons of results available in literature and using the multi-purpose software package Abaqus.

2. A flat shell finite strip

The displacement components of a flat shell finite strip are approximated as a series of products of the polynomials u_{0m}, v_{0m} and w_m and trigonometric functions Y_m and Y_{vm}:

$$u_0(x, y) = \sum_{m=1}^{r} u_{0m}(x)Y_m(y) \qquad v_0(x, y) = \sum_{m=1}^{r} v_{0m}(x)Y_{vm}(y)$$
$$w(x, y) = \sum_{m=1}^{r} w_m(x)Y_m(y), \tag{1}$$

where r is a number of series terms, u_0, v_0 and w are the displacement