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Analysis of the distortion of cantilever box girder with inner flexible diaphragms using initial parameter method

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ABSTRACT

In this paper, the distortion of cantilever box girders with inner flexible thin diaphragms is investigated under concentrated eccentric loads using initial parameter method (IPM), in which the in-plane shear strain of diaphragms is fully considered. A high-order statically indeterminate structure was established with redundant forces, where the interactions between the girder and diaphragms were indicated by a uniform distortional moment. Based on the compatibility condition between the girder and diaphragms, solutions for the distortional angle and the warping function were obtained by using IPM. The accuracy of IPM was verified by finite element analysis for the distortion of cantilever box girders with 2, 5 and 9 diaphragms under three diaphragm thicknesses, respectively. Taking a lifting mechanism as an example, parametric studies were then performed to examine the effects of the diaphragm number and thickness, the ratio of height to span of the girder, the hook's location and the wheels' positions on the distortion of cantilever box girders. Numerical results were summarized into a series of curves indicating the distribution of distortional warping stresses and displacements for various cross sections and loading cases.

1. Introduction

Cantilever box girders are widely used as the main load bearing structural components in many cases. For instance, at container seaports, cantilever cranes are applied to handle cargos from the boat to port (Fig. 1a). In construction process, precast bridge segments are elevated and installed by cantilever cranes (Fig. 1b). For cantilever girders subjected to eccentric loads, the flexure, torsion and distortion of the cross section are commonly concerned by designers. Both warping deformations and stresses produced by distortional loads are usually so large that it may have significant values in comparison with the torsional and flexural ones.

In order to control the distortion of the beam cross section, diaphragms are installed at the interior of girders, which can increase not only the stability of local plate, but also the resistance to warping deformation and stresses [1,2]. The primary research on the distortion of girder has been performed using two methods – the Beam on Elastic Foundation (BEF) analogy [3] and the Equivalent Beam on Elastic Foundation (EBEF) analogy [4,5], where a thin diaphragm is analogous to simple supports and a thick solid diaphragm to fixed supports. Additionally, the effect of shear strain of the cross section on distortion is considered in EBEF analogy, and cannot be ignored when the frame shear stiffness is significant to distortional warping one for box girders [6,7]. Since there is no clear boundary between the thin and thick diaphragms, it is difficult to accurately estimate the deformation and stresses of beams in BEF and EBEF methods when considering the thickness of diaphragms.

For a cantilever box girder with inner diaphragms, the key analysis is the interactions between the girder and diaphragms. A high-order statically indeterminate structure can be modeled for girders with inner diaphragms under eccentric loads, where the interactions are indicated by redundant forces and moments [8–10], both are obtained from the finite strip method [11] and the force method [8]. This model has been extensively researched on multi-span curved beams [12,13]. Besides, an extended trigonometric series method [14] is applied to girders with inner diaphragms, where the thin-plate theory is applied. Interactions between the girder and diaphragms are indicated by compatibility conditions with respect to both displacements and forces. This method can achieve a high accuracy for both displacement and stresses, but the number of simultaneous equations is so large even for girders with few diaphragms that it is difficult to apply in practice. For example, there are up to 720 simultaneous equations for a girder with only two

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Nomenclature

A, C	top and bottom flanges
B, D	right and left webs
$B_d(z)$	distortional bimoment of cross section z
b,h	width and height of girder
E	Young's elastic modular
G	shear modular
H_{ij}, V_{ij}	inner horizontal and vertical redundant forces
$H(\alpha)$	unit step function of variable α
I_t, I_k, I_c	warping/polar/frame moment of inertia
l	span of girder
M, N, J, K	four angle nodes
$M_d(z)$	distortional moment of cross section z
M_j	distortional moment produced by jth loads P_j
M_{pi}	distortional moment for ith diaphragm
m, n	total number of loads and diaphragms
m_d	distributed distortional moment
n_1,n_2	distance between point O and webs
O	original point
P_j	jth concentrated load
$\boldsymbol{P}(z)$	transfer matrix in IPM
R, S	total number of diaphragms and loads before the calculated point z
s	circumferential coordinate around profile
t_1 t_2	thickness of left and right webs
t_3	thickness of flanges
t_{pi}	thickness of ith diaphragm
v	Possion's ratio
W_{add}	the additional distortional warping function
$W(z)$	distortional warping function
x,y	in-plane coordinate axes of cross section
z	longitudinal axis of girder
z_j	location of jth concentrated load P_j
z_{pi}	mid-line position of ith diaphragm
$\boldsymbol{Z}(z)$	state vector of cross section z in IPM
β_d	ratio of warping stresses between nodes J and N
γ_{pi}	in-plane shear strain of ith diaphragm
φ_1, φ_2, φ_3, φ_4	combinations of trigonometric function
λ_1, λ_2	distortional coefficients of girder
θ	torsional angle of cross section
$\chi(z)$	distortional angle of cross section z
χ_{add}	the additional distortional angle
τ_d	distortional shear stress
$\boldsymbol{\Phi}(z)$	initial transfer matrix in IPM
$^{(1)}$, $^{(2)}$, $^{(3)}$, $^{(4)}$, $^{(i)}$, $^{(j)}$	first, second, third, fourth, ith and jth differentiates

diaphragms [14].

Finite element analysis (FEA) is another method used to analyze distortion of the girder with inner diaphragms. The influence of the diaphragm number on the deformation and stresses has been investigated for straight [15–17], curved [18–20] and multi-cell [21,22] box girders with diaphragms by using FEA, where diaphragms were presumed to possess infinite in-plane shear stiffness and free warping for both torsion and distortion. The assumption of infinite shear stiffness does not fit for girders with flexible thin diaphragms. Considering the finite in-plane shear stiffness of diaphragms, a distortional stiffness ratio is introduced [23] which is the stiffness of thin or thick diaphragms to that of a solid-plate diaphragm. Both the type and location of diaphragms will make an influence on the horizontal loading distribution and to a less extent on the vertical one [24–26]. Research shows that orthogonal diaphragms are superior to skewed ones in reducing transversal bending stresses [27] and arranging the lateral loading distribution [28].

The initial parameter method (IPM), originally introduced to solve the non-uniform torsion of beams by Vlasov [29], has been extended to analyze the distortional deformation and stresses. In IPM, either the distortional angle or the warping function was taken as the original variable in the distortion equation [30–32], and the distortional deformation and stresses can be obtained according to the boundary conditions. High accuracy for both deformations and stresses produced from IPM has been verified by FEA on girders without diaphragms. However, IPM has not been extensively applied to girders with inner diaphragms. In addition, interactions between the girder and its flexible thin diaphragms are still not clear.

Previous researches on girders with inner diaphragms has been generally performed under the assumption of an infinite in-plane shear (distortional) stiffness, where the in-plane shear deformation of diaphragms was totally restrained and the out-of-plane warping deformation was free [15–22]. However, this assumption is not applicable to girders with flexible thin diaphragms [15,18]. The main objective of this work is to analyze the distortion of cantilever girders with inner flexible thin diaphragms under eccentric loading, where the in-plane shear deformation of diaphragms is fully considered. Considering the compatibility between the girder and diaphragms, solutions for both the distortional deformations and stresses are obtained by using IPM. Numerical results are verified by applying FEA. Finally, taking a lifting mechanism as an example, a series of parametric studies are performed to examine the effects of the number and thickness of diaphragms, the hook's location and the positions of trolley wheels on the distortion of cantilever girder with inner flexible diaphragms.

(a) gantry crane (b) bridge construction

Fig. 1. Examples of cantilever girders.