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# Nonlinear dynamic analysis and vibration of eccentrically stiffened S-FGM elliptical cylindrical shells surrounded on elastic foundations in thermal environments

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A B S T R A C T

Elliptical cylindrical shell is one of shells with special shape. Up to date, there is no publication on vibration and dynamic of functionally graded elliptical cylindrical shells. Therefore, the purpose of the present study is to investigate the nonlinear dynamic response and vibration of imperfect eccentrically stiffness functionally graded elliptical cylindrical shells on elastic foundations using both the classical shell theory (CST) and Airy stress functions method with motion equations using Volmir's assumption. The material properties are assumed to be temperature - dependent and graded in the thickness direction according to a Sigmoid power law distribution (S-FGM). The S-FGM elliptical cylindrical shell with metal-ceramic-metal layers are reinforced by outside metal stiffeners. Both the S-FGM elliptical shell and metal stiffeners are assumed to be in thermal environment and both of them are deformed under temperature simultaneously. Two cases of thermal loading (uniform temperature rise and temperature variation through thickness) are considered. The nonlinear motion equations are solved by Galerkin method and Runge-Kutta method (nonlinear dynamic response, natural frequencies). The effects of geometrical parameters, material properties, elastic foundations Winkler and Pasternak, the nonlinear dynamic analysis and nonlinear vibration of the elliptical cylindrical shells are studied. The some obtained results are validated by comparing with those in the literature.

## 1. Introduction

Cylindrical shells are frequently used in the manufacturing of aircrafts, missiles, boilers, automobiles, pipelines and some submarine structures. Furthermore, because of the main components of metal and ceramic with physical properties which are changed with the change in thickness, FGM structures have a very high mechanical strength and heat resistance, therefore, vibration and dynamic analysis of FGM cylindrical shells are one of the major issues that many researchers are especially interested in.

Sheng and Wang [1] studied the thermal vibration buckling and dynamic stability of functionally graded cylindrical shells embedded in an elastic medium. Duc considered nonlinear thermal dynamic analysis of eccentrically stiffened S-FGM circular cylindrical shells [2] and eccentrically stiffened piezoelectric S-FGM circular cylindrical shells [3] surrounded on elastic foundations using the higher-order shear deformation shell theory. Ng et al. [4] investigated the dynamic stability analysis of FGM cylindrical shells under periodic axial loading. Bahadori and Najafizadeh [5] presented the free vibration analysis of two-dimensional functionally graded axisymmetric cylindrical shell on Winkler–Pasternak elastic foundation by first-order shear deformation theory and using Navier-differential quadrature solution methods. Bich and Nguyen [6] proposed the nonlinear vibration of functionally graded circular cylindrical shells based on improved Donnell equations. Shariyat [7] investigated the dynamic buckling of suddenly loaded imperfect hybrid FGM cylindrical shells with temperature-dependent material properties under thermo-electro-mechanical loads. Du et al. [8] considered the nonlinear forced vibration of functionally graded cylindrical thin shells. Sofiyev and Kuruoglu [9,10] investigated the buckling and vibration of shear deformable functionally graded orthotropic cylindrical shells under external pressures and the dynamic instability of three-layered cylindrical shells containing an FGM interlayer. Duc and Thang [11] studied the nonlinear response of imperfect eccentrically stiffened ceramic-metal-ceramic FGM thin circular cylindrical shells surrounded on elastic foundations and subjected to axial compression. Song et al. [12] investigated the active vibration control of CNT-reinforced composite cylindrical shells via piezoelectric patches. Shen [13] studied the large amplitude vibration behavior of

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a shear deformable FGM cylindrical shell of finite length embedded in a large outer elastic medium and in thermal environments. Kandasamy et al. [14] considered the numerical study on the free vibration and thermal buckling behavior of moderately thick functionally graded structures in thermal environments. Sepiani et al. [15] investigated the free vibration and buckling of a two-layered cylindrical shell made of inner functionally graded and outer isotropic elastic layer, subjected to combined static and periodic axial forces. Kadoli and Ganesan [16] presented the buckling and free vibration analysis of functionally graded cylindrical shells subjected to a temperature-specified boundary condition. Sepiani et al. [17] studied the vibration and buckling analysis of two-layered functionally graded cylindrical shell, considering the effects of transverse shear and rotary inertia. Jafari et al. [18] proposed the nonlinear vibration of functionally graded cylindrical shells embedded with a piezoelectric layer. Mehralian et al. [19] studied the size-dependent formulation of shear deformable functionally graded piezoelectric cylindrical nano shells is developed based on a new modified couple stress theory. In [20], Duc et al. also considered the nonlinear dynamic analysis of Sigmoid functionally graded circular cylindrical shells on elastic foundations using third order shear deformation theory in thermal environments.

Elliptical cylindrical shell which is one of special cylindrical shell forms also attracts the researchers' attention. Yue et al. [21] studied the elliptical crack normal to functionally graded interface of bonded solids. Ganapathi et al. [22] investigated the free flexural vibration behavior of laminated angle-ply elliptical cylindrical shells. Tornabene et al. studied free vibrations of composite oval and elliptic cylinders by the generalized differential quadrature method in [23] and presented dynamic analysis of thick and thin elliptic shell structures made of laminated composite materials in [24]. Khalifa. [25] considered the effects of non-uniform Winkler foundation and non-homogeneity on the free vibration of an orthotropic elliptical cylindrical shell. Gholizadeh et al. [26] investigated the non singular material parameters for arbitrarily elliptical-cylindrical invisibility cloaks. Li el al. [27] studied the prediction of the elastic critical load of submerged elliptical cylindrical shell based on the vibro-acoustic model. Shariati and Rokhi [28] proposed the numerical and experimental investigations on buckling of steel cylindrical shells with elliptical cutout subject to axial compression. Ahmed [29] investigated the buckling behavior of a radially loaded corrugated orthotropic thin-elliptic cylindrical shell on an elastic foundation.

However, there are very few publications about buckling and post buckling as well as vibration of elliptical plates and elliptical cylindrical shells made of FGM materials. In 2005, Patel et al. [30] presented the free vibration characteristics of functionally graded elliptical cylindrical shells using finite element procedure and the higher-order theory including variable transverse displacement through the thickness. In 2013, Zhang [31] proposed the nonlinear bending analysis of FGM elliptical plates resting on two-parameter elastic foundations. Recently, Duc et al. [32] presented on the nonlinear buckling and postbuckling of an eccentrically stiffness S-FGM elliptical cylindrical shells in thermal environment. And according to the authors' knowledge, no paper on dynamic analysis for FGM elliptical cylindrical shells is published so far.

Therefore, this paper set a research objective of researching the nonlinear dynamic response and vibration of an imperfect eccentrically stiffened functionally graded elliptical cylindrical shells in thermal environment. The material properties are assumed to be temperature - dependent and graded in the thickness direction according to a Sigmoid power law distribution in terms of the volume fractions of constituents with metal - ceramic - metal layers (ES-S-FGM shells). One surface of the ES-S-FGM shells is reinforced by outside metal stiffeners. The S-FGM elliptical cylindrical shells are reinforced by longitudinal and transversal stiffeners and surrounded by Winkler and Pasternak type elastic foundations. Both properties of S-FM elliptical cylindrical shells and stiffeners are assumed to be temperature dependent and deformed under temperature simultaneously. The governing equations are established based on CST theory with motion equations using Volmir's assumptions. The time-amplitude response curves of the cylindrical shell are obtained and the effects of excitation force, elastic foundations, stiffeners, geometrical parameters, material properties, imperfections, mechanical and thermal loads on the vibration and nonlinear dynamic response of ES-S-FGM shells are examined (Fig. 1).

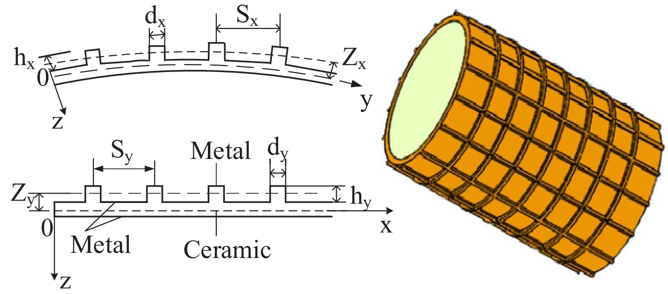


**Fig. 1.** Geometry and the coordinate system of the functionally graded elliptical cylindrical shells with metal-ceramic-metal layers reinforced by (ES-S-FGM).

## 2. Modeling of the ES-S-FGM elliptical cylindrical shells surrounded on elastic foundations

Consider an eccentrically stiffened moderately thin elliptical cylindrical shells with metal-ceramic-metal layers (ES-S-FGM elliptical cylindrical shells). The length, mean radius and total thickness of the shell are $L$, $R$ and $h$, respectively. The outside of the ellipse is eccentrically stiffeners in both directions (where $s_x$, $s_y$ are spacing of the stringer and ring stiffeners, respectively; $A_x$, $A_y$ are cross-section areas of stiffeners; $z_x$, $z_y$ are eccentrically of stiffeners with respect to the middle surface the shell; $d_x$, $h_x$ and $d_y$, $h_y$ are width and height of the stringer and ring stiffened, respectively) and the inner is placed on the elastic foundations. The shell is defined in a coordinate system $(x, \theta, z)$ where $x$ and $\theta$ are in the axial and circumferential directions of the shell, respectively, and $z$ is perpendicular to the surface and points outwards $(-h/2 \le z \le h/2)$.

### 2.1. Material properties of the ES-S-FGM shells

Because FGM are typically made from a mixture of metal and ceramic, their material properties are related to both the material properties and the continual distribution of the constituent materials [33–35,37]. Meanwhile, the material properties of both metal and ceramic are related to environmental temperature. Thus, FGM material properties vary smoothly through their thickness and exhibit temperature dependency.

The material properties of the constituent materials $P_r$ (where the subscripts "$r$" will be replaced with "$c$" or "$m$" corresponding to ceramic or metal, respectively) are usually expressed as the following nonlinear function of temperature $T$ [32,38]:

$$P_r(T) = c_0(c_{-1}T^{-1} + 1 + c_1T + c_2T^2 + c_3T^3), \quad (1)$$

in which $T = T_0 + \Delta T$, $\Delta T$ is the temperature increment of the environment containing the shell and $T_0 = 300K$ (room temperature), and $c_0$, $c_{-1}$, $c_1$, $c_2$, $c_3$ are coefficients characterizing of the constituent materials with temperature-dependent given in Table 1.

The material properties of FGM such as the elastic modulus $E$, the mass density $\rho$ and the thermal expansion coefficient $\alpha$ are related not only to the material properties of the constituent materials, but also to their volume fraction $V_c$ and $V_m$:

$$P(z, T) = P_c(T)V_c(z) + P_m(T)V_m(z), \quad V_c + V_m = 1 \quad (2)$$

in which $P_c$ and $P_m$ denotes a material property of ceramic and metal.

For an S-FGM shell made of two different constituent materials with metal-ceramic-metal layers, the volume fractions $V_c(z)$ and $V_m(z)$ can be