Thin-Walled Structures 117 (2017) 282–293

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Full length article

End condition effect on distortional buckling of cold-formed steel columns with arbitrary length

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ARTICLE INFO

Keywords:
Cold-formed steel column
Distortional buckling
Analytical formulae
End condition

ABSTRACT

Usual design practice for distortional buckling considers a lower bound solution as the actual buckling load. In reality, this practice is inconsistent with actual case since the obtained buckling load is a constant value no matter how long the column is and whatever the end condition is. According to available literature, the research dealt with such a problem is found quite rare. In this scenario, this paper presents an analytical approach to establish a new distortional buckling formula, which takes both the effects of column length and end condition into consideration. The formula was derived based on an edge stiffened plate model. The model was assumed to be pin-ended and fix-ended so as to investigate their effects. The Galerkin method was employed to derive the distortional buckling formula. Further, simplifications to the rigorous formula were made to allow them to be easily used by the engineers. Subsequently, in order to verify the accuracy of the derived formula, the results obtained from the derived formula were compared with the numerical results obtained from the computer software GBTUL. In addition, the performance of the derived formula was further verified by comparing the corresponding ultimate strength based on Shafer's DSM expressions with numerical result from the literature. The comparison and validation result shows that the derived formula (i) can be used successfully in estimating the distortional buckling load for both pin-ended and fix-ended columns with practical length and (ii) can general more rational buckling strength estimation due to the consideration of column length and end condition effect.

1. Introduction

Advances in manufacturing and low production costs have prompted the Cold-Formed Steel (CFS) industry to search for more structurally efficient cross-section shapes. One of the most favorable ways to perform this task is adding edge stiffeners to form an edge stiffened cross-section, as shown in Fig. 1. The edge stiffened cross-sections are increasingly popular in CFS structures due to their ease of fabrication and superior strength-to-weigh ratios. However, on the other hand, a complicated cross-section instability phenomenon, named distortional buckling [1], was found to often govern the structural response and failure of the edge stiffened members with intermediate lengths. For this reason, extensive efforts were devoted by many researchers to investigate distortional buckling behavior of cold-formed members.

According to Lau and Hancock's research [2], the distortional buckling of the edge stiffened cross-sections can be modeled by using an equivalent stiffened bar model. Taking C-section as an example, Fig. 2(a) and (b) presented the cross-sectional buckling deformation and the analysis model, respectively. In Fig. 2(b), the flange and the lip together were treated as an angle section with no local (plate) buckling (i.e., only global buckling develops.), while the element interactions of web on the flanges were modeled by translation springs and rotational spring. Schafer and Pekoz [3] investigated the accurate influence of web buckling on the rotational spring stiffness (k_φ) by using an interaction-buckling model between the web and flange. The study presented a theoretical method to consider the influence of web buckling on the distortional buckling instead of the empirical reduction factors for stress in the web employed in Lau and Hancock's method [2]. Teng and Yao [4] studied the distortional buckling behavior of channel beam-column under axial compression and biaxial bending. The solution of the study can be considered as an extension of Lau and Hancock from axially loaded columns to beam-columns. Li and Chen [5] modified Lau and Hancock's model to consider the adverse effect of flexure deformation of the flange itself. In the study, the position of the translation spring in Fig. 2 was placed at the centroid of the compression flange and lip system. Recently, Huang and Zhu [6] proposed a new stiffened plate model for calculating the critical distortional

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http://dx.doi.org/10.1016/j.tws.2017.04.029
Received 1 March 2017; Received in revised form 20 April 2017; Accepted 26 April 2017

Nomenclature

A_g	the gross cross-section area
b	width of flange
$C\gamma$	correction factor for boundary condition
C_w	correction factor for flexure deformation of flange itself
D	plate flexural rigidity per unit width = $Et^3/12(1\text{-}\nu^2)$
d	width of lip
E	Young's modulus
G	shear modulus of elasticity = $E/2(1+\nu)$
g_1, g_2, g_3	the functions of m
h	depth of web
I_x, I_y	second moments of area of the edge stiffened plate model about the x- and y-axes
I_{xy}	product second moment of the area of edge stiffened plate model about the x-and y-axes
I_w	warping constant of the edge stiffened plate model
J	torsion constant of the edge stiffened plate model
k_x	stiffness of lateral restraint
k_φ	stiffness of rotational restraint
L	length of column
m	number of half-wavelength
m_{cr}	critical number of half-wavelength
P	distortional buckling load
Q_y	intensity of reaction force along the elastic support acting in the y-direction
t	plate thickness
u, v, φ	deflections in the x- and y-directions of the shear-center axis and angle of rotation of section about that axis
x_0, y_0	x- and y-coordinates of the shear center
h_x,h_y	x- and y-distance of centroid from the flange/web junction
λ	critical half-wavelength determined by software GBTUL
υ	Poisson's ratio
σ_{GBTUL}	distortional buckling stress obtained by software GBTUL [22] considering end condition and column length effects
σ_p	distortional buckling stress obtained by the proposed formula
$\sigma_{s,GBTUL}$, σ_{Lau}	distortional buckling stress obtained by software GBTUL [22] and Lau and Hancock's formula [2] with assuming the columns being buckled in one half-wavelength and being pin-ended, respectively

buckling load of cold-formed C-section columns. In the study, the web is treated as a plate which has both in-plane and out of plane bending deformations, whereas the flange and lip are modeled as the angle stiffener applied at the two ends of the web, which is subjected to asymmetric bending and torsion. The model does not involve any springs but the critical stress can be calculated analytically. Apart from the aforementioned analytical methods, finite strip methods [7–13], finite element methods [14–16] and generalized beam theory (GBT) [17–21] have also been used to study the distortional buckling of edges stiffened columns and beams.

It is worth to mention that, based on GBT theory, Silvestre and Camotim [20] studied the effects of column length and end conditions on distortional buckling stresses of cold-formed C- and Z-section columns. The study presented valuable information concerning the column length and end condition effects on distortional behavior of cold-formed C- and Z-section columns. Further, in order to quantify these effects on distortional buckling stresses, general GBT-based fully analytical formulae were derived by the authors. However, it should be pointed out that the presented formulae are too complicated, which makes them cannot be solved without resorting to computer programs.

Following above literature review, it can be found that research concerning the distortional buckling of edge stiffened columns has been conducted widely. However, in order to facilitate analytical derivations, nearly all the aforementioned analytical approaches (apart from the formulae in Ref. [20]) assumed the column being buckled in only one half-wavelength and pin-ended. In addition, free warping condition is considered. Although the above assumptions may help to reduce derivation works, the obtained buckling load is a constant value no matter how long the column is and whatever the boundary condition is. In order to clearly illustrate the above problem, Fig. 3 shows the relationship between the lengths, the end conditions and the distortional buckling stress for a C-section column, where the result in the figure was obtained by using the software GBTUL [22]. As it can be observed in Fig. 3, the practical distortional buckling stresses σ_p (represented by the dash black solid line for fix-ended condition and red dash line for pin-ended condition) are clearly different with the obtained distortional buckling load σ_a (represented by the blue dash line) based on the aforementioned artificial assumption. This result proves the importance of end conditions and column length on the distortional buckling behavior of columns. In addition, as it can be seen in Fig. 3, the value of the distortional buckling stress σ_p is also sensitive to the end conditions for short column, where the stress difference between the pin-ended and fix-ended condition is clearly significant. It is worth to mention that Zhou et al. [23] studied the distortional buckling behavior of fix-ended and pin-ended C-section columns. This study also proved that the distortional buckling load of columns can be significantly affected by the end conditions. However, in order to simplify the derivation works, the column in Ref. [23] was also artificially assumed to be buckled in one half-wavelength for both pin-ended and fix-ended conditions, which is unsuitable for columns if

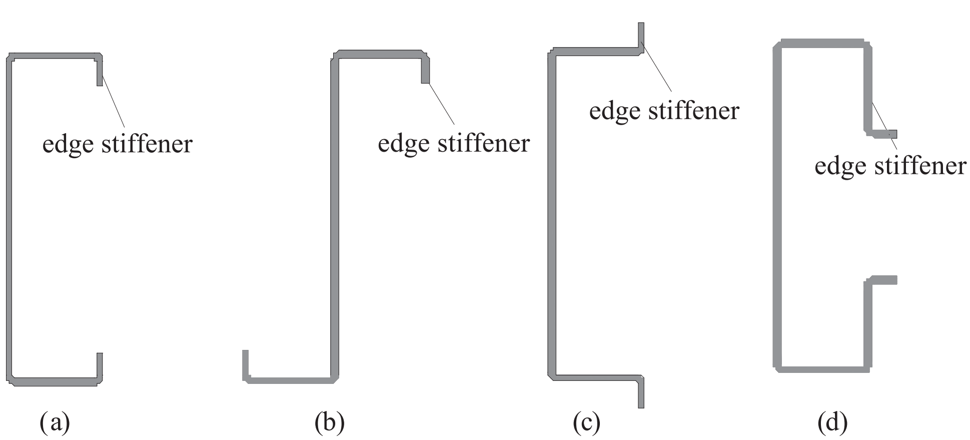

Fig. 1. Edge stiffened cross-sections: (a) C-section. (b) Z-section. (c) hat-section. (d) rack-section.