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Buckling analysis of stiffened plate structures by an improved meshfree flat shell formulation

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A B S T R A C T

An efficient Galerkin meshfree flat shell formulation is presented for the analysis of buckling behaviors of stiffened plate structures. Both plate bending and membrane deformations are approximated by the reproducing kernel particle method (RKPM). The governing equation is transformed into a weak form, and it is discretized by the scattered nodes. The stiffness matrix is numerically integrated with the nodal integration technique, i.e., the stabilized conforming nodal integration (SCNI). The RKPM and SCNI based flat shell modeling approach can address the shear locking problem. Additionally, the present discretization is further improved by involving a drilling rotation component, which is to effectively model the stiffeners. There are six degrees of freedom per node. A singular kernel is also introduced into a set of the interpolants to model the web/flange connection, as well as the imposition of the essential boundary conditions. A generalized eigenvalue problem is analyzed for evaluating buckling loads/modes of the stiffened plate structures. The accuracy of the numerical results and the effectiveness of the proposed method are examined through several numerical examples.

1. Introduction

A ship's hull structure is subjected to longitudinal bending induced by external loads, e.g., self-weight, cargo weights and wave forces. The hull structure is generally composed of plates, stiffeners and stiffened plate structures [1–4]. It is important to design the structural members optimally, by choosing the thickness and aspect ratio of the plating as well as size of the web/flange and the number of stiffeners to prevent the occurrence of the structural failures within a limited construction expense. Many researches have been performed on the evaluation of buckling loads/modes and found the ultimate strength for the stiffened plate and hull structures, e.g., see Refs. [5–10]. The present study focuses on buckling analysis of the plate and stiffened plate structures, by employing a novel numerical simulation method.

In recent years, meshfree and other related methodologies, e.g., the element free Galerkin method (EFGM) [11], the reproducing kernel particle method (RKPM) [12], the extended finite element method [13,14], the isogeometric analysis [15–18], and the wavelet Galerkin method [19–23], have been widely adopted to analyze scientific and engineering problems. Meshfree method is particularly attractive for the analysis of plate and shell structures. Continuous functions can be used to approximate the deflection and rotational components, smooth stress/strain distributions are obtained throughout the entire analysis domain, and the shear locking problem can also be avoided. Krysl and Belytschko [24,25] analyzed plate and shell problems by employing the EFGM. Noguchi et al. [26] solved shell and spatial structures by EFGM employing a convected coordinate system. Kanok-Nukulchai et al. [27] examined the shear locking property of meshfree plate bending problems. Generally, plate and shell problems with flat or smoothly curved surfaces were modeled by the meshfree method because a special treatment is required to address the displacement discontinuity or its derivative. Zhang et al. [28] analyzed shell structures with discontinuities employing a moving least-square approximation with a discontinuous derivative function. Tanaka et al. [29–31] treated the displacement discontinuity of a cracked shear deformable plate by modifying the reproducing kernel (RK) interpolation functions along

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the crack segment with a diffraction method and a visibility criterion [32,33].

An efficient Galerkin meshfree method is presented here to analyze buckling behaviors of assembled plate structures such as a stiffened plate structure. A flat shell formulation is employed based on the RK approximation [12,34–37]. Mindlin-Reissner plate formulation is adopted to represent the plate bending deformation, and plane stress condition is assumed for the membrane deformation. The in-plane and out-of-plane deformations are coupled and approximated by the RKs. The flat shell is modeled by the scattered nodes, and the stabilized conforming nodal integration (SCNI) [38,39] is employed to accurately integrate the stiffness matrices with the Voronoi cell diagram [40]. The flat shell modeling on the basis of the RKPM and SCNI can overcome the shear locking problem by imposing a so-called Kirchhoff mode reproducing condition (KMRC) [41,42]. Wang and Sun [43] and Sadamoto et al. [44] analyzed geometrically nonlinear problems for the flat shells. Yoshida et al. [45] succeeded in producing a linear buckling analysis of a flat shell model including curved stiffeners. The flat shell modeling involves five degrees of freedom (5DOFs) per node, and the stiffeners were modeled by suppressing the deflection components of the flat shell model. Curved shell problems were also analyzed in [46].

In the present research, a drilling rotation component is included to model stiffeners efficiently. Therefore, the meshfree discretization possesses six degrees of freedom (6DOFs) per node. Because the drilling component does not have any resisting force or stiffness, a penalty energy function proposed by Kanok-Nukulchai [47] is introduced. In the author's previous study, a multiple point constraint (MPC) technique was adopted for the meshfree web/flange modeling and imposition of the essential boundary conditions in [48]. However, stress oscillation was found along the boundary conditions in the MPC enforcements [44]. A singular kernel (SK) [49] is then applied to impose the so-called Kronecker delta function property in the set of the meshfree interpolants. Additionally, sub-domain stabilized conforming integration (SSCI) [50–55] is employed for evaluating the stiffness matrices around the web/flange connections. So far, research has been conducted to analyze the buckling behaviors of the plates and assembled plate structures using meshfree and related methods in [56–66]. The modeling of stiffened plate structures based on RKPM and SCNI, and high accuracy buckling loads/modes evaluations have not been reported yet. The mathematical formulation and discretization of the proposed method are presented for analyzing the stiffened plate structures. The calculated results are critically examined through the numerical examples.

The contents of this paper is as follows. The meshfree flat shell formulation including the drilling rotation component and the nodal integration techniques are presented in Section 2. Modeling of the stiffened plate structures is discussed in Section 3. Numerical examples for several buckling problems for plate and stiffened plate structures are presented in Section 4. Conclusions are given in Section 5.

2. Meshfree modeling for a flat shell

2.1. Governing equations for linear buckling analysis

When simulating buckling behaviors of plate structures, the plate bending deformation, in contrast to the membrane deformation, cannot be neglected. A flat shell formulation is developed by combining the in-plane and out-of-plane deformations. A schematic flat shell model is represented in Fig. 1. S is the area of plate and t_h is the plate thickness. A plane stress condition and Mindlin-Reissner plate theory are adopted to allow shear deformation of the plate. The membrane deformations in the x_1- and x_2-directions at the mid-thickness plane are represented by $u_{1\mathrm{mid}}$ and $u_{2\mathrm{mid}}$, respectively, the deflection component of the plate is represented by u_3, and the rotational angles for the x_1- and x_2-axes are represented by θ_1 and θ_2, respectively. θ_3 is the drilling rotation component.

The plate deformation $\boldsymbol{u}(\boldsymbol{x})$ can be expressed as:

$$\boldsymbol{u}(\boldsymbol{x}) = \begin{Bmatrix} u_1(\boldsymbol{x}) \\ u_2(\boldsymbol{x}) \\ u_3(\boldsymbol{x}) \end{Bmatrix} = \begin{Bmatrix} u_{1mid}(\boldsymbol{x}) + z\theta_2(\boldsymbol{x}) \\ u_{2mid}(\boldsymbol{x}) - z\theta_1(\boldsymbol{x}) \\ u_3(\boldsymbol{x}) \end{Bmatrix}, \tag{1}$$

where $u_k(\boldsymbol{x})$ $(k=1,2,3)$ are components of the displacement toward the x_k-axes. $z(|z| \le t_h/2)$ represents the distance from the mid-thickness plane of the plate.

When considering an elastic stability problem of a shear deformable plate, the following weak form can be obtained:

$$\int_V \boldsymbol{\sigma}:\, \delta\boldsymbol{\varepsilon}_L dV + \lambda \int_V \boldsymbol{\sigma}'_0:\, \delta\boldsymbol{\varepsilon}_{NL} dV \;=\; 0, \tag{2}$$

where $\boldsymbol{\varepsilon}_L$ and $\boldsymbol{\varepsilon}_{NL}$ are the linear and nonlinear strain tensors. δ represents variational operator. V is the volume of the plate. The linear strain components ε_{Lij} $(=\boldsymbol{\varepsilon}_L)$ can be described as:

$$\begin{Bmatrix} \varepsilon_{L11} \\ \varepsilon_{L22} \\ 2\varepsilon_{L12} \\ 2\varepsilon_{L31} \\ 2\varepsilon_{L23} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_{1mid}}{\partial x_1} + z\dfrac{\partial \theta_2}{\partial x_1} \\ \dfrac{\partial u_{2mid}}{\partial x_2} - z\dfrac{\partial \theta_1}{\partial x_2} \\ \dfrac{\partial u_{1mid}}{\partial x_2} + \dfrac{\partial u_{2mid}}{\partial x_1} + z\left(\dfrac{\partial \theta_2}{\partial x_2} - \dfrac{\partial \theta_1}{\partial x_1}\right) \\ \dfrac{\partial u_3}{\partial x_1} + \theta_2 \\ \dfrac{\partial u_3}{\partial x_2} - \theta_1 \end{Bmatrix}. \tag{3}$$

The nonlinear strain tensor $\boldsymbol{\varepsilon}_{NL}$ is defined as:

$$\varepsilon_{NLij} \;=\; \frac{1}{2}\left(\frac{\partial u_k}{\partial x_i}\frac{\partial u_k}{\partial x_j}\right). \tag{4}$$

Additionally, $\boldsymbol{\sigma}=\{\sigma_{11}\ \sigma_{22}\ \sigma_{12}\ \sigma_{31}\ \sigma_{23}\}^T$ is the Cauchy stress tensor. The stress-strain relationship can be written as $\boldsymbol{D}\boldsymbol{\varepsilon}_L$, and the elastic coefficient matrix $\boldsymbol{D}$ for the shear deformable plate is written as:

$$\boldsymbol{D} \;=\; \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 & 0 & 0 \\ \nu & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1-\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & \kappa\frac{1-\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & \kappa\frac{1-\nu}{2} \end{bmatrix}, \tag{5}$$

where κ is the shear correction factor, and $\kappa=\pi^2/12$ is adopted. E is the Young's modulus and ν is the Poisson's ratio. $\boldsymbol{\sigma}'_0$ is a pre-buckling stress tensor which is represented as:

$$\boldsymbol{\sigma}'_0 \;=\; \begin{bmatrix} \sigma'_{0\,11}\boldsymbol{I} & \sigma'_{0\,12}\boldsymbol{I} & \sigma'_{0\,13}\boldsymbol{I} \\ \sigma'_{0\,21}\boldsymbol{I} & \sigma'_{0\,22}\boldsymbol{I} & \sigma'_{0\,23}\boldsymbol{I} \\ \sigma'_{0\,31}\boldsymbol{I} & \sigma'_{0\,32}\boldsymbol{I} & \boldsymbol{0} \end{bmatrix}, \tag{6}$$

where $\boldsymbol{I}$ is a 3×3 unit tensor. $\sigma'_{0\,33}$ is set as zero based on the plane stress assumption.

2.2. A RKPM meshfree approximation of a flat shell

In the meshfree discretization, nodes are distributed on the mid-thickness plane of the plate as shown in Fig. 1. The physical quantities are approximated by a linear combination of the RK functions. Each node has 6DOFs, i.e., three in-plane deformation components $(u_{1mid}, u_{2mid}, \theta_3)$ and three out-of-plane deformation components $(u_3, \theta_1, \theta_2)$, respectively. The 6DOFs are denoted as: $\{u_{1mid}\ u_{2mid}\ u_3\ \theta_1\ \theta_2\ \theta_3\}^T = \{u_1\ u_2\ u_3\ u_4\ u_5\ u_6\}^T$. The vector components $u_i^h(\boldsymbol{x})$ $(i=1, \ldots, 6)$ $(=\boldsymbol{u}^h(\boldsymbol{x}))$ are represented by RK $\psi_I(\boldsymbol{x})$ $(I=1, \ldots, NP)$ as:

$$u_i^h(\boldsymbol{x}) \;=\; \sum_{I=1}^{NP}\psi_I(\boldsymbol{x})u_{iI}, \quad (i=1, \ldots, 6), \tag{7}$$