Thin-Walled Structures xxx (xxxx) xxx–xxx

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Model reduction in thin-walled open-section composite beams using variational asymptotic method. Part I: Theory☆

Dineshkumar Harursampath[a], Ajay B. Harish[b,*], Dewey H. Hodges[c]

[a] Department of Aerospace Engineering, Indian Institute of Science, Bangalore 560012, India
[b] Institute of Continuum Mechanics, Leibniz Universität Hannover, Appelstr. 11, Hannover 30167, Germany
[c] Daniel Guggenheim School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA 30332, USA

ARTICLE INFO

Keywords:
Anisotropic
Asymptotic
Beam
Bending
Cantilever
Composite materials
Fibre reinforced
Laminated
Stochastic
Warping
Thin-walled
Open-section
Variational Asymptotic Method
Trapeze effect
Vlasov effect
Model reduction

ABSTRACT

This two-part work describes the development of a comprehensive and reliable tool for analysis of the most commonly used geometries of thin-walled, open-section composite beams. Part one describes formulation of an asymptotically-correct reduced order model and simple validation examples. The model, developed using the mathematically rigorous Variational Asymptotic Method (VAM), is capable of capturing all nonlinear and non-classical effects observed in anisotropic beams. It leads to closed forms solutions and thus rapid, yet accurate, analysis. Part two describes the application of developed theory to most commonly used geometries of thin-walled, open-section composite beams.

1. Introduction

Thin-walled composite beams have become an integral part of many engineering structures today. The technology, which evolved mainly from the aerospace industry (e.g., rocket motor casings, space booms, rotor blades, etc.), finds its application in fields as diverse as sports (e.g., skis, tennis rackets, golf clubs, etc.), sea hulls, offshore rigs, automotive components (e.g., power transmission shafts), and architecture (e.g., I-section beams, channel-section beams, etc.). Despite their superior engineering properties and enhanced manufacturing technology, their development is not up to their potential due to cost considerations. One of the sure ways to reduce development cost is to establish accurate analysis tools to aid in tailoring of composite structures. In particular, analytical tools that can be used in preliminary design and efficient numerical tools that can be used in detailed design are of prime importance.

Existing classical analytical tools suffice for most simple structures. However, many important practical phenomena have been observed in thin-walled beams to which these classical tools are blind. The effects being dealt with in this work are non-classical nonlinear effects. We begin with the introduction and definitions of some terminology.

Physical nonlinearities are basically nonlinearities in the stress-strain relationship becoming important by virtue of strain being large. Geometrical nonlinearities, on the other hand, are nonlinearities in the strain-displacement relationship becoming important by virtue of rotation being (at least moderately) large. A nonlinear beam theory could arise either due to 1-D physical nonlinearities or 1-D geometric nonlinearities or both. Certain 1-D physical nonlinearities occur because of 3-D geometric nonlinearities (large 3-D warping), and these are called non-classical nonlinearities. Composite open-section beams have low torsional rigidity and hence allow fairly large twist rates, not only due to torsion but also to coupling with other types of loading that accentuate the importance of nonlinearities involving twist. These effects are important in the case of thin-walled beams, but in an asymptotic sense they are not so important in the cases of beams with solid or thick-walled sections. Summarizing, in slender structures

☆ A part of this paper was published at 49th AIAA Structures, Structural Dynamics, & Materials Conference, Schaumburg, IL, USA, Apr 7–10 (2008).

* Corresponding author.
E-mail addresses: dinesh@aero.iisc.ernet.in (D. Harursampath), harish@ikm.uni-hannover.de (A.B. Harish), dhodges@gatech.edu (D.H. Hodges).

http://dx.doi.org/10.1016/j.tws.2017.03.018
Received 28 October 2015; Received in revised form 18 December 2016; Accepted 14 March 2017

Nomenclature

x_1	Cartesian coordinate along the reference axis of a beam
x_i	Cartesian coordinates for a cross section, $i = 2, 3$
$\mathbf{b}_i$	unit vectors for undeformed geometry, $i = 1, 2, 3$
$\mathbf{B}_i$	unit vectors for deformed geometry, $i = 1, 2, 3$
q_i	rigid-body-like displacements of a cross section, $i = 1, 2, 3$
Γ_{ij}	3-D strains, $i, j = 1, 2, 3$
σ_{ij}	3-D stresses, $i, j = 1, 2, 3$
E^{ijkl}	3-D stiffness constants, $i, j, k, l = 1, 2, 3$
$\epsilon_{\alpha\beta}$	2-D membrane strains, $\alpha, \beta = 1, 2$
$\rho_{\alpha\beta}$	2-D elastic curvatures, $\alpha, \beta = 1, 2$
A_{ij}	2-D membrane stiffness constants, $i, j = 1, 2, 6$
B_{ij}	2-D coupling stiffness constants, $i, j = 1, 2, 6$
D_{ij}	2-D bending/twisting stiffness constants, $i, j = 1, 2, 6$
γ_{11}	1-D axial strain
κ_1	1-D elastic twist per unit length
κ_i	1-D elastic bending curvatures, $i, j = 2, 3$
S	1-D stiffness matrix
$\overline{w}_i$	warping of a cross section, $i = 1$ (out-of-plane), $i=2, 3$ (in-plane)
ℓ	characteristic wavelength of a beam
b	characteristic dimension of a cross section
h	thickness of a thin wall
k_1	initial twist per unit length of beam
k_2	initial flatwise curvature of a strip
δ_h	small parameter, $\frac{h}{b}$
δ_b	small parameter, $\frac{b}{\ell}$
δ_t	small parameter, bk_1
ϵ	small parameter, magnitude of largest Γ_{ij}
N_s	total number of strips making up an open-section beam
N_j	total number of joints in an open-section beam
$N_c(m)$	number of strips connected at m-th joint

undergoing motions with large wavelengths, these nonlinear and non-classical effects may play a significant role. Moreover, additional refinements are required to enhance the accuracy of the solution in the following cases: (a) Timoshenko refinement is required when short wavelength modes associated with transverse shear are involved; (b) Vlasov refinement is required for thin-walled, open-section beams, as captured in the current work; and (c) general refinements are required when new degrees of freedom are introduced particular to the problem – such as high-frequency vibrations.

The current work is motivated by the need for a general-purpose tool to capture the non-classical effects of thin-walled, open-section composite beams in closed-form via an asymptotic method. It is focused at developing analytical solutions incorporating nonlinear strain fields, which are the sources of many such effects. This work addresses non-classical effects observed in thin-walled, open-section beams that can be considered as a pretwisted assembly of strips. The primary 3-D sources of nonlinearity in such a beam stem from the out-of-plane cross-sectional warping. This 3-D nonlinearity manifests as two well-known non-classical effects, namely Trapeze and Vlasov effects.

Before proceeding ahead, it is important to understand the classification of beams as discussed in [26]. As per the nomenclature used in this work, beams are classified into T-class, S-class and R-class. Consider the characteristic length along the thickness is h and along the cross-sectional plane is a. T-class refers to the thin-walled, open-section beams that is of primary importance to this work. T-class beams are torsionally soft, in comparison to bending, since the strain induced by twisting is $O(h\kappa_1)$, while the strain induced by bending is $O(a\kappa_2)$ or $O(a\kappa_3)$. S-class beams are strip-like with $a >> ha >> h$. These beams are soft in torsion and also in one bending direction. High aspect-ratio wings, helicopter blades etc fall into this category. Finally, the R-class beams are the regular beams that are neither T-class nor S-class and do not demonstrate significant length effects. Some examples are closed-cell, solid, closed-section beams.

Thin-walled composite beams are widely favored in aerospace structures. Examples of sections that can be analyzed using the present work include I-sections, T-sections, X-sections, Z-sections, cruciforms, and any custom section that can be constructed from an assembly of arbitrarily oriented straight elements. Though 3-D finite element modeling of such beams is possible, it is computationally expensive when modeling large structures. Existing classical analytical tools suffice for most simple structures. However, many important practical phenomenon have been observed in thin-walled composite as well as isotropic beams to which classical tools are blind. Many existing 1-D models make *ad hoc* assumptions for thin-walled beams and neglect the non-classical effects.

This work takes advantage of the beam-like configuration, which allows consideration of the ratio between a characteristic cross-sectional dimension and the wavelength of the deformation along the beam as a small parameter. The non-linear 3-D problem is decomposed into two simpler problems: a two-dimensional (2-D) nonlinear analysis, which provides in a compact form the cross-sectional properties using a mathematical technique called the Variational-Asymptotic Method (VAM), and a nonlinear one-dimensional (1-D) problem along the length of the beam. The results from the former analysis act as inputs to the latter, and both may be highly nonlinear. Owing to this decomposition, a major simplification ensues in the complex 3-D problem. This efficient approach to the structural analysis of beams is outlined as a flowchart in Fig. 1.

The nonlinear cross-sectional analysis provides stiffness coefficients along with recovery relations, which are functions of the generalized 1-D strain measures. The stiffness coefficients are needed to solve the 1-D beam problem. Solving the 1-D problem, one obtains the 1-D displacements and generalized strain measures. After both problems have been solved, the in-plane and out-of-plane warping of the cross section, the total 3-D displacement field, and the 3-D strain and stress fields can all be recovered.

VAM also brings into importance the concept of asymptotic correctness. The theoretical or numerical solutions obtained are approximate in nature. Accuracy of the numerical solutions are asserted by reduction in relative error norms (or otherwise often referred to as

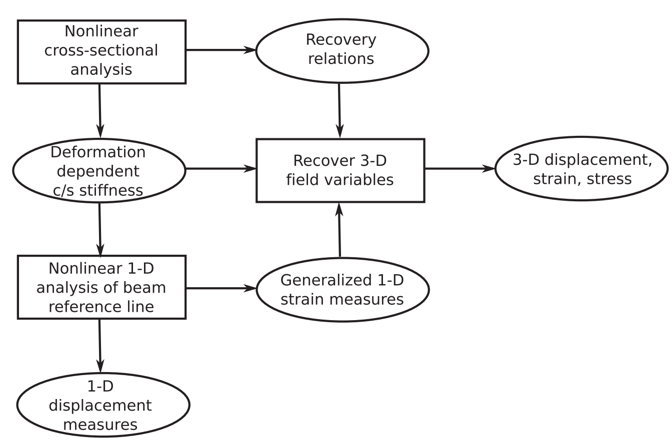

Fig. 1. Procedure for beam analysis.