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Estimation of maximum torsional moment for multicorner tubes

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ABSTRACT

In this paper, the estimation of maximum torsional moment for multicorner tubes under torsional loading was investigated using nonlinear FE analysis. The effects of tube geometries and strain-hardening coefficient on the torsional behaviour were discussed. The maximum torsional moment was due to the occurrence of sectional collapse of the tubes, and the mechanism of sectional collapse could be classified into three physical phenomena, elastic buckling, plastic yielding and plastic flattening of the cross section. Moreover, based on our numerical results, analytical solution for estimating the maximum torsional moment for each phenomenon was proposed.

1. Introduction

Thin-walled circular and multicorner tubes have been widely used in many structural applications as lightweight structural components. Once the external load is applied to the tubes, the external work is dissipated by the elastic and plastic strain energy due to axial tension or compression, bending and torsion.

Over the decades, the application of such tubes as impact energy absorbers for vehicles has been investigated by many researchers [1–6]. For example, Alexander [1] proposed his theoretical model to estimate the mean crushing load of circular tubes subjected to axial compressive load. Also, Abramowicz and Wierzbicki [2] developed their theoretical analysis to evaluate the plastic resistance of rectangular and multicorner tubes under axial compression. When the tubes with moderate length are subjected to axial load, the first peak load which associates with local plastic buckling are found in the early stage of deformation, and the following load fructuates due to the wrinkle's folding behaviour. The absorbed energy can be estimated by the load-displacement curve after plastic buckling took place. Therefore, the estimation of average compressive load under such load fructuation is significant for the energy absorption capacity for tubes.

On the contrary, when the tubes are subjected to bending or torsional load, the reaction bending or torsional moment increases and reaches a peak, and then drops dramatically and never recover due to the occurrence of sectional collapse. Therefore, the estimation of maximum bending or torsional moment for tubes is important to estimate its load-carrying capacity.

The elastic and plastic collapse behaviour of thin-walled tubes under torsion have also been published by many researchers [6–11]. For example, Murray [6] investigated the maximum torsional load for thin-walled square tubes, and proposed a solution to distinguish the load caused by elastic buckling with that caused by plastic yielding. In Murray's research, the effect of strain-hardening was neglected [6]. Also, Chen and Wierzbicki [9], Chen et al. [10] and Chen [11] studied the torsional collapse of thin-walled square and circular columns analytically and numerically. They mainly discussed the collapse behaviour of square column under large plastic rotation. In their study, they proposed three-phase of collapse mechanism, namely, pre-buckling, post-buckling and collapse-spreading, and developed their solution to evaluate the torsional moment versus rotation curve for each phase using energy method. Also, they extended their solution for rectangular and hexagonal thin-walled columns, and compared analytical and numerical results. Their analytical results agreed fairly with numerical results, however, there were still large differences between these results, especially the maximum torsional moment. This implies that their solution is limited to be applied in square tubes, and that other modified analytical development is still needed for estimating the torsional moment of multicorner tubes. Basically, the maximum torsional moment and deformation behaviour for tubes depend strongly on the tube geometries and the amount of strain-hardening. The estimation technique should be constructed by observing the variation of stress states and deformation behaviour in details.

The main purpose of our study is to evaluate the maximum torsional moment for multicorner tubes by using nonlinear FE analysis. The appearance of maximum torsional moment is due to the occurrence of sectional collapse of the tubes, and the mechanism of sectional collapse could be classified into three physical phenomena, elastic buckling, plastic yielding and plastic flattening of the cross-section. Based on the

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Table 1
List of material parameters.

Young's modulus E [GPa]	72.4
Poisson's ratio	0.3
Yield strass σ_Y [MPa]	72.4
Hardening coefficient E_h [MPa]	$E/100 \le E_h \le E/10$

investigation of stress variation and cross-sectional distortion in our FE analysis, the analytical solutions for estimating the maximum torsional moment for each phenomenon are proposed.

2. Method of numerical analysis

In our study, the commercial finite element analysis software, MSC.Marc, is used to demonstrate the torsional deformation behaviour of multicorner tubes. The multicorner tubes are assumed to be isotropic, homogeneous elastoplastic material, and obey the bilinear hardening curve as shown in Eq. (1) where E_h shows the strain-hardening coefficient.

$$\sigma = \sigma_Y + E_h\left(\varepsilon - \frac{\sigma_Y}{E}\right). \tag{1}$$

Values of the material properties applied in our FE model are as shown in Table 1 unless it is stated. As for the finite element mesh, the 2-dimensional shell element is used for the thin-walled tubes with 5 mm × 5 mm mesh size. also in our calculation, the mises' yield criterion and the updated lagrangian method are used for formulating the nonlinear problem, and the newton-raphson numerical method is applied for finding the root effectively.

The boundary condition and tube's geometry of our FE model are explained as follows. All rotations and displacements except for the movement in the axial direction U_x is fixed at one end of the tube, and the torsional rotation θ is applied at the other end of the tube via an attached rigid plate. Fig. 1(a) shows the schematic of boundary condition, and Fig. 1(b) portrays three types of cross-sectional shape of multicorner tubes discussed in our paper. The longitudinal coordinate sets x, while y and z denote the coordinate of the cross-section in each direction. Here, the wall's width b and the tube's length L are set to 50 mm and 200–250 mm respectively, and the thickness t varying from 0.35 to 4.0 mm. The tube's length L may affect the torsional response of tubes, but our FE results confirmed that when the length-to-width ratio L/b is larger than 4.0, the effect can be negligible. In this paper, the estimation of maximum torsional moment of square tubes is mainly investigated analytically and numerically, while the validation of the proposed theoretical equations on the multicorner tubes is summarized in Section 3.4.

3. Results and discussion

It is well known that for the torsional response of hollow straight tubes, the sectional collapse would occur when the end rotation reaches a certain angle. Once the sectional collapse occurs, the torsional moment reaches a maximum and drops considerably. The occurrence of sectional collapse has two reasons:

- the shear stress distributed over the cross-section reaches a limit
- the large amount of plastic deformation, namely, plastic flattening deformation arises

The former includes two physical phenomena, namely, the occurrence of elastic buckling and plastic yielding, and these phenomena can be observed in relatively thin-walled tubes. In the following, the torsional moment due to elastic buckling and plastic yielding are written by $M_{col}{}^e$ and $M_{col}{}^p$ respectively. On the contrary, as for the latter case, the walls of the tubes move inward and the applied moment drops significantly even when the shear stress still increases after the collapse. This phenomenon can be observed in relatively thick-walled tubes. Here the torsional moment due to plastic flattening is written by $M_{col}{}^f$. Fig. 2(a)–(c) show examples of the torsional moment M_T versus torsional angle θ (M_T-θ) diagram of square tubes having different tube's thickness-to-width ratio t/b and different strain-hardening coefficient E_h/E. Also in these figures, two kinds of stress variation are plotted by dotted and dashed lines. Here, τ_A represents the shear stress at point 'A' where the maximum deflection takes place, and τ_{ave} represents the average shear stress at the same position $x = x_A$. It can be found in these

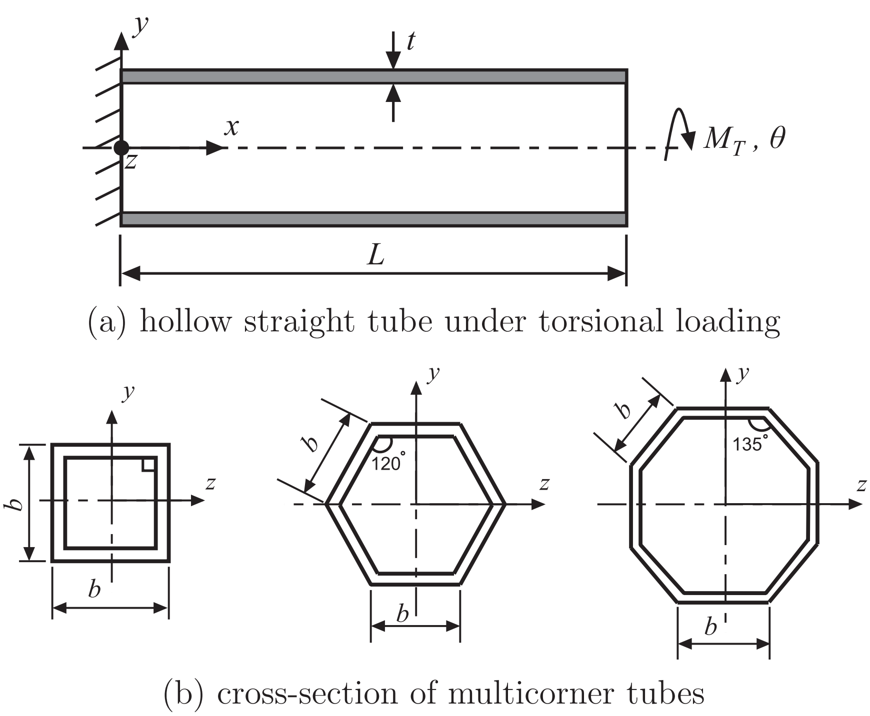

Fig. 1. Multicorner tubes subjected to torsional loading. (a) Hollow straight tube under torsional loading, (b) cross-section of multicorner tubes.