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Buckling and vibrations of metal sandwich beams with trapezoidal corrugated cores – the lengthwise corrugated main core

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ABSTRACT

The paper is devoted to the stability of an orthotropic multi-layered beam. This beam is an untypical sandwich structure the faces of which consist of three layers. The original mathematical model of the beam is formulated taking into account different properties of each layer. From the Hamilton's principle the system of equations of motion is derived which is the base for the analysis of buckling and vibration problems. As a result of the analysis the buckling load and natural frequencies of exemplary plates have been obtained. The results are compared with these given by the numerical solution realised with the use of the finite element method in the ANSYS and ABAQUS systems.

1. Introduction

The theory of sandwich structures is developed from the middle of 20th century, that is evidenced by the papers published in journals and at conferences. Basic facts on sandwich structures, their generalizations and applications, can be found in, e.g., Libove, Hubka [1], Allen [2] and Ventsel, Krauthammer [3]. Besides monographs there are also many papers on this subject. Computational models for sandwich plates and shells, predictor-corrector procedures, and the sensitivity of the sandwich response to variations in the different geometric and material parameters have been studied by Noor, Burton and Bert [4]. Carlsson, Nordstrand, Westerlind derived tension, shear, bending and twisting rigidities for sandwich structures with corrugated core. The paper [5] is devoted to the computation of the effective properties of corrugated core sandwich panels. Talbi, Batt, Ayad, Guo [6] presented an analytical homogenization model for corrugated cardboard and its numerical implementation in a shell element. In the paper [7] by Cheng, Le, Lu the finite element method (FEM) is used to derive equivalent stiffness properties of sandwich structures with various types of cores. Similar results can be found in [8,9].

The present work is a continuation of the research on multi-layered structures with corrugated core conducted by the authors and co-authors. First works [10,11] were derived to classical sandwich structures. Strength and stability of aluminium beams with lengthwise and crosswise corrugated core have been analysed. To increase the flexural stiffness of such beams the modification has been introduced presented in works [12,15]. In structures presented here the faces are composed of two elements: inside one which is a corrugated plate and outside one in the form of a flat sheet. The corrugations of the core and the faces are perpendicular to each other. The results of experiments on these five-layered beams presented in papers [16,17] showed that although the stiffness of the beam is much higher when compare to the stiffness of a three-layered beam, the connection between two corrugated plates can be the weak point of the structure. Further modification has been made then by introducing a flat sheet between two corrugated plates. This way a seven-layered beam has been obtained – a sandwich beam with three-layered faces, as can be seen in Fig. 1.

The thin-walled beam presented in this paper is an innovatory orthotropic structure, not referred to in the literature. The main core is a lengthwise corrugated sheet. The two faces are three-layered structures the core of which, referred to as face core, is a crosswise corrugated sheet. The internal and external sheets of the faces are flat. All layers of the beam are made of the same material which is isotropic and homogeneous. A characteristic feature of the beam consists in differentiation of shear effects in particular layers, according to the core's corrugation direction. The deformation of the beam's cross section also depends on this direction. An original mathematical model of the structure will be formulated in the following sections. The model will include the hypothesis of deformation of the cross-section as well as rigidities of the layers in particular directions.

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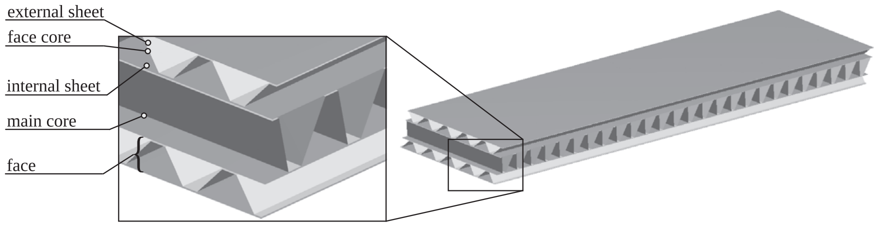

Fig. 1. Scheme of the beam with lengthwise corrugated core.

2. Analytical studies

2.1. Mathematical model of the beam

The classical approach to modelling of sandwich structures is to assume a broken line hypothesis as to the field of displacements. For more than three layers the zig-zag hypotheses can be used formulated by Carrera [18]. In the proposed structure the stiffness of the core of the faces is considerably higher than the stiffness of the main core. For this reason it is assumed that the three-layered faces deforms according to Kirchhoff-Love hypothesis and the shear effect is present in the main core only. Consequently, the field of displacements of the cross section of the beam takes the form shown in Fig. 2. Introducing dimensionless function describing displacements $\psi(x) = u_1(x)/t_{c1}$ it can be expressed as follows:

- the upper facing $-(\frac{1}{2}t_{c1} + 2t_s + t_{c2}) \le z \le -\frac{1}{2}t_{c1}$

$$u(x, z) = -z\frac{dw}{dx} - u_1(x), \tag{1}$$

- the lower facing $\frac{1}{2}t_{c1} \le z \le \frac{1}{2}t_{c1} + 2t_s + t_{c2}$

$$u(x, z) = -z\frac{dw}{dx} + u_1(x), \tag{2}$$

- the lengthwise corrugated main core $-\frac{1}{2}t_{c1} \le z \le \frac{1}{2}t_{c1}$

$$u(x, z) = -z\left[\frac{dw}{dx} - 2\psi(x)\right] \tag{3}$$

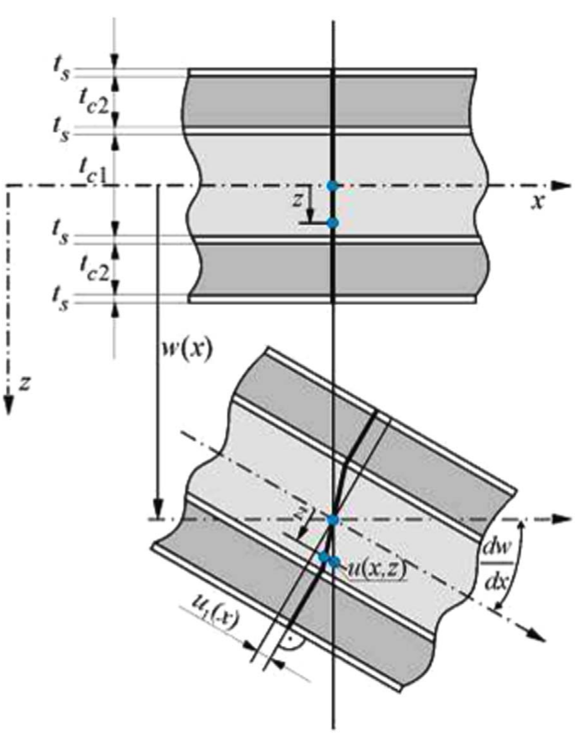

Fig. 2. Scheme of deformation of beam's cross section.

In the above formulae t_{c1}, t_{c2}, t_s (see Fig. 2) describes the thicknesses of particular layers. The indexes $c1$, $c2$ and s corresponds to the inner core, the core of the faces and flat sheets, respectively. For the assumed hypothesis of deformation of the cross section – no shear effect in the faces – the geometric relations, the strains, are

$$\varepsilon_x^{(s)} = \frac{\partial u}{\partial x}, \quad \gamma_{xz}^{(s)} = 0, \quad \varepsilon_x^{(c2)} = \frac{\partial u}{\partial x}, \quad \gamma_{xz}^{(c2)} = 0, \ \varepsilon_x^{(c1)} = \frac{\partial u}{\partial x},$$
$$\gamma_{xz}^{(c1)} = \frac{\partial u}{\partial z} + \frac{dw}{dx} = 2\psi(x). \tag{4}$$

Since the stiffness of each layer is different, depending on the geometry, the physical relations, according to Hooke's law, have to be written in the form

$$\sigma_x^{(s)} = E\varepsilon_x^{(s)}, \quad \tau_{xz}^{(c1)} = G_{xz}^{(c1)}\gamma_{xz}^{(c1)}, \quad \sigma_x^{(c1)} = E_x^{(c1)}\varepsilon_x^{(c1)}. \tag{5}$$

Stiffness moduli of the main core can be expressed as $E_x^{(c1)} = \widetilde{E}_x^{(c1)}E$ and $G_{xz}^{(c1)} = \widetilde{G}_{xz}^{(c1)}E$ where $\widetilde{E}_x^{(c1)} = \frac{x_{b1}x_{01}^3}{2(\widetilde{s}_{a1} + x_{f1}x_{b1})}$ is dimensionless Young's modulus in which $\widetilde{s}_{a1} = [(1 - x_{01})^2 + x_{b1}^2(\frac{1}{2} - x_{f1})^2]^{1/2}$, $x_{01} = \frac{t_{01}}{t_{c1}}$, $x_{f1} = \frac{b_{f1}}{b_{01}}$, $x_{b1} = \frac{b_{01}}{t_{c1}}$ (indexes 01 and 02 corresponds to the main core and the inner corrugated layer of facings, respectively); $\widetilde{G}_{xz}^{(c1)}$ is dimensionless shear modulus described in details in [19]. E is the Young's modulus of the material of the beam. The Hamilton's principle can be written as follows:

$$\delta\int_{t_1}^{t_2}[T - (U_\varepsilon - W)]dt = 0 \tag{6}$$

where
$T = \frac{1}{2}\int\int_V\int\rho(\frac{\partial w}{\partial t})^2 dV$ – the kinetic energy, $U_\varepsilon = \frac{1}{2}\int\int_V\int(\sigma_x\varepsilon_x + \tau_{xz}\gamma_{xz})dV$ – the elastic strain energy,
$W = \frac{1}{2}\int_0^L F_0(\frac{\partial w}{\partial x})^2 dx$ – the work of the load, t_1, t_2 – the initial and final times,
ρ – the mass density of the beam, L – the length of the beam, F_0 – the compressive force.

Based on the above principle the equations of motion have been derived

$$\begin{cases} bt_{c1}c_\rho\rho_s\cdot\frac{\partial^2 w}{\partial t^2} + Ebt_{c1}^3\left[2c_{ww}\frac{\partial^4 w}{\partial x^4} - c_{w\psi}\frac{\partial^3\psi}{\partial x^3}\right] = -F_0\frac{\partial^2 w}{\partial x^2} \\ c_{w\psi}\frac{\partial^3 w}{\partial x^3} - 2c_{\psi\psi}\frac{\partial^2\psi}{\partial x^2} + 4\frac{\psi(x)}{t_{c1}^2}\widetilde{G}_{xz}^{(c1)} = 0 \end{cases} \tag{7}$$

where