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On dimensionless numbers for predicting large ductile transverse deformation of monolithic and multi-layered metallic square targets struck normally by rigid spherical projectile

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ABSTRACT

The principal objective of this study is to develop two empirical equations based on new dimensionless numbers for predicting large ductile transverse deformation of monolithic and multi-layered square targets due to a normal impact of a rigid spherical projectile. In the first step, a complete set of dimensionless parameters is generated based on the Buckingham π-theorem for nondimensionalization of the governing equations of quadrangular plates subjected to dynamic loading. New dimensionless numbers are suggested based on the dimensionless governing equations of the plate where consider the effect of the geometry of structural member and projectile, material strength and density, impact velocity of the projectile, and strain rate sensitivity of materials. The dimensionless numbers are taken into account according to the input and output experimental data pairs. Lastly, nonlinear relationships between the ratios of maximum permanent deflection to target thickness and dimensionless numbers are established by using a novel mathematical approach, namely, singular value decomposition method. The outcomes of the empirical equations compare well with the experimental results on the single and layered targets made of either mild steel or aluminum alloy or a combination of them and illustrate good agreement with experiments for different impact velocities ranging from 42 to 158 m/s. Hence, it seems that the methodology of the present study can readily be employed to derive straightforward closed-form equations for complicated processes where several experimental input and output data pairs are existing.

1. Introduction

In recent years, one of the complex problems in solid mechanics is impact and it has converted to an attractive topic for many scholars [1–14]. Production engineers and designers are interested in impact engineering because of its usages at hole-flanging and high-velocity blanking processes. Also, aerospace researchers require comprehending the process of penetration for designing structures which are more useful versus the projectile impact. This science also has been used by vehicle manufacturers for improving the safety and performance of their productions.

Penetration of projectiles into ferrous or nonferrous targets with different nose shapes is one of the impact issues which can be made more difficult because of cutting and plugging phenomena [15–18]. The Plugging phenomenon happens when a section of the target is removed from the vicinity of impact edges due to the cutting [19]. The penetration process of a flat projectile has been described at three stages: first, primary target elongation without cutting at impact edges, second, penetration and plug elongation, and third, plug separation from the target [20]. In 1983 [21], the researchers used the theory of plastic stress wave propagation for analyzing the penetration process. In this analysis, the process has been divided into five steps. Later on, in 1987 [22], a new structural model based on the theory of plastic shear deformation was presented for thin plates due to impact loads. It should be noted that all of the mentioned solutions were presented based on the assumption of the rigid projectile. In the recent past, Li and Chen [23] performed a dimensional analysis for penetration of non-deformable projectiles into plain and reinforced concrete targets and it concluded that merely two dimensionless numbers, namely, the geometry function of projectile and the impact function, as compositions of dominant physicals factors, specify the final penetration depth in concrete targets due to impact loads. Dey and her colleagues [24] studied both experimentally and numerically, the ballistic perforation resistance of 12 mm thick monolithic and double-layered targets made of Weldox 700 E steel plates against impact loads. In these series of experiments, different target combinations were impacted by ogival

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and blunt projectiles, and the ballistic limit velocity was calculated for each sample. The comparison between experimental results and numerical simulations demonstrated good agreements. The main conclusion of this study can be expressed as follows; (1) for the case of when blunt projectiles was used, the double-layered systems offered a large gain in the ballistic limit velocity, and (2) the overall protection level was obtained without depending on the projectile nose shape and also was considerably increased by double-layering the target. Borvik and his co-workers [25] investigated the ballistic perforation resistance of different steel materials with Lagrange discretization and compared against each other. In these experiments, the yield strengths were changed ranging from 600 to 1700 MPa. According to the European norm EN1063, two different ballistic protection classes, namely, BR6 and BR7 were considered. In both classes, the impact velocity was 830 m/s. The experimental results and FE simulations showed a linear correlation between yield strength and ballistic limit velocity. By using the explicit finite element code LS-DYNA, Flores-Johnson and his colleagues [26] numerically studied the ballistic performance of monolithic and multi-layered plates impacted normally by a 7.62-mm APM2 projectile, where the impact velocity varied in the range of 775–950 m/s. The specimens were made of either aluminum or mild steel or a combination of these materials. The obtained results can be expressed as follows; (1) for the specimens made of same material, monolithic targets showed better ballistic performance than multi-layered ones and this effect diminished with impact velocity, (2) for the similar areal density, double-layered targets with a thick back steel plate and thin front aluminum plate represented greater resistance than multi-layered steel targets when the areal density is similar for these specimens. The influences of using multi-layered plates instead of monolithic ones have been studied in the open literature for the last decade and it can be deduced that the experimental data on large ductile transverse deformation of multi-layered is contradictory and limited. Hence, detailed experimental and empirical works are still required.

Due to the complex essence of this topic, the presented analytical studies were not considered all multiple physical phenomena. Hence, empirical modeling based on input and output experimental data pairs can be used as an alternative method to better understand the behavior of complicated systems. Furthermore, development of empirical models can assist the scholars for validation of their experimental and analytical results as well as the engineers to abstain from performing more repetition experiments at the design process [27].

Taking into consideration of the aforesaid investigations, the principal objective of this paper is to present empirical equations for predicting maximum permanent transverse deflections of monolithic and multi-layered metallic square targets due to the normal impact of a rigid spherical projectile. Hence, a complete set of dimensionless parameters is defined based on Buckingham-π theorem and dimensional analysis for nondimensionalization of the governing equations of quadrangular plates due to dynamic loads. Then, the empirical equations are obtained by proposing appropriate dimensionless numbers through the dimensionless governing equations as well as applying a novel mathematical approach. In order to evaluate the accuracy of empirical equations performance, the results of obtained models are compared with a large number of impact tests results on monolithic and multi-layered metallic square targets were conducted by using a single stage gas gun apparatus.

2. Empirical modeling

In this section, new empirical constitutive equations based on dimensional analysis and Buckingham-π theorem are presented by proposing appropriate input and output dimensionless numbers for predicting maximum transverse deflections of monolithic and multi-layered metallic targets due to spherical projectile impact. The dimensionless numbers are obtained through the dimensionless governing equations of square plates due to dynamic loads and it can be concluded that they have obvious physical meanings. These numbers are taken into account based on input and output experimental data pairs and then a novel mathematical approach, namely, singular value decomposition method is applied in conjunction with suggested numbers to extract the empirical models.

2.1. Dimensionless numbers

The classical plate theory is applied for suggesting some appropriate dimensionless numbers for estimating maximum transverse deflections of monolithic and multi-layered metallic square targets impacted normally by a spherical projectile. The experimental results of this study have such large displacements relative to the plate thicknesses. Hence, the effect of membrane forces should be considered in the analysis. According to Kirchhoff-Love theory, the governing equations of thin quadrangular plates against dynamic loading are written as following equations [28–31].

$$\left(\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2}\right) + \left(N_x\frac{\partial^2 w}{\partial x^2} - 2N_{xy}\frac{\partial^2 w}{\partial x \partial y} + N_y\frac{\partial^2 w}{\partial y^2}\right) = \rho H\frac{\partial^2 w}{\partial t^2} - P \tag{1}$$

Where w, ρ, H and P are the deflection, material density, thickness of the structural member and dynamic pressure, respectively. In these equations, M_x and M_y are bending moments per unit length, M_{xy} is torsion moments per unit length, Q_x and Q_y are transverse shear per unit length, N_x, N_y and N_{xy} are membrane forces per unit length. By applying Buckingham-π theorem, a complete set of dimensionless parameters is defined as $m_x = M_x/M_0$, $m_{xy} = M_{xy}/M_0$, $m_y = M_y/M_0$,$n_x = N_x/N_0$, $n_{xy} = N_{xy}/N_0$, $n_y = N_y/N_0$, $N_0 = \sigma_0 H$, $M_0 = \sigma_0 H^2/4$, $W = w/H$, $X = x/L$,$Y = y/L$ and $T = C_s t/H$. In these dimensionless numbers, C_s is sound velocity of environment, M_0, N_0 and σ_0 are fully plastic bending moment, fully plastic membrane force and mean flow stress, respectively (Please see Ref. [30] for more details). Now by introducing dimensional analysis approach, as well as considering the defined dimensionless parameters to the basic governing equations of plates, a dimensionless governing equation of fully clamped square plates is obtained as follows.

$$\left(\frac{\partial^2 m_x}{\partial X^2} + 2\frac{\partial^2 m_{xy}}{\partial X \partial Y} + \frac{\partial^2 m_y}{\partial Y^2}\right) + 4\left(n_x\frac{\partial^2 W}{\partial X^2} - 2n_{xy}\frac{\partial^2 W}{\partial X \partial Y} + n_y\frac{\partial^2 W}{\partial Y^2}\right) = 4\left(\frac{L}{H}\right)^2\left(\frac{E}{\sigma_0}\frac{\partial^2 W}{\partial T^2} - \frac{P}{\sigma_0}\right) \tag{2}$$

In the above equation, L and E are half-length of the structural member and elastic modulus.

By using Cowper-Symonds equation and Jones' assumption [31,32] for the mean strain rate $\dot{\varepsilon}$, the influence of strain-rate sensitivity of materials can be added to the dimensionless governing equation as follows.

$$\left(\frac{\partial^2 m_x}{\partial X^2} + 2\frac{\partial^2 m_{xy}}{\partial X \partial Y} + \frac{\partial^2 m_y}{\partial Y^2}\right) + 4\left(n_x\frac{\partial^2 W}{\partial X^2} - 2n_{xy}\frac{\partial^2 W}{\partial X \partial Y} + n_y\frac{\partial^2 W}{\partial Y^2}\right) = 4\left(\frac{L}{H}\right)^2\left(\frac{E}{\sigma_d}\frac{\partial^2 W}{\partial T^2} - \frac{P}{\sigma_d}\right) \tag{3}$$

Where

$$\dot{\varepsilon} = \frac{W_0 V_0}{3\sqrt{2}BL} \tag{4}$$

$$\frac{\sigma_d}{\sigma_0} = 1 + \left(\frac{\dot{\varepsilon}}{D}\right)^{\frac{1}{q}} = 1 + \eta\left(\frac{W_0}{H}\right)^{\frac{1}{q}} \tag{5}$$

So, a new dimensionless number $\eta = \left(\frac{V_0 H}{3\sqrt{2}BLD}\right)^{\frac{1}{q}}$ is suggested for