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Differential quadrature thermal buckling analysis of general quadrilateral orthotropic auxetic FGM plates on elastic foundations

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ABSTRACT

Majority of the available research on buckling analysis of the plates, has been devoted to plates with well-behaved configurations, e.g., rectangular or circular geometries. In the present research, thermal buckling of general quadrilateral plates fabricated from heterogeneous, orthotropic, and auxetic (with negative Poisson ratio) materials resting on elastic Winkler-Pasternak elastic media is investigated. Edges of the plate may be either simply supported or clamped. Thus, the problem is a quite general one, from the material, boundary conditions, and to some extent, geometry points of view and may cover wide ranges of the practical applications, as special cases. The stability equations are derived through transformation of the governing equations of the plate from the geometric rectangular Cartesian coordinates to the computational natural coordinates and discretization of the resulting equations by means of the differential quadratic method. Buckling analysis has been accomplished through investigation of the pre-buckling and buckling onset situations. Finally, effects of the skew angles of the general quadrilateral plate, heterogeneity index, orthotropy angle, edge condition, foundation stiffness, and auxeticity of the material on the buckling temperature rises are investigated comprehensively.

1. Introduction

Plates with quite well-behaved geometries, e.g., rectangular plates, are usually employed in the simple engineering structures only. Infrastructures and underframes of the aerospace, mechanical, vehicular, rail, and marine vehicles are generally constructed from curved plates (shells) or plates with either curved or generally non-parallel edges. Therefore, investigation of buckling behavior of the skew plates is of practical importance. Experimental and numerical observations reveal that significant compressive stresses that may lead to buckling may be induced in the plates, due to small temperature rises in the surrounding environment [1–6] and sometimes, buckling sensitivity to the thermal loads may be more pronounced than that of the direct mechanical loads, especially for thinner plates.

The local excessive stresses that either lead to delamination or slippage of the fibers relative to the matrix of the traditional orthotropic plates, due to using distinct phases and layers of materials, may be eliminated by using functionally graded materials (FGMs) [7,8]. However, orthotropy of the material properties is advantageous when a directional stiffening is crucial. Orthotropic functionally graded materials and constructions may be achieved through using a large number of perfectly bonded thin orthotropic layers with slightly varying material properties [9–14]. The heterogeneity of the materials may also stem from effects of the humidity or moisture [6,9,10]. While vibration and buckling of the orthotropic FGM shells were first investigated by Sofiyev et al. [9] and Sofiyev [10–12], mechanical/thermal buckling of the FGM orthotropic plates has been investigated by Asemi and Shariyat [13], Shariyat and Asemi [14], and Mansouri and Shariyat [6,15]. The elastic foundation alters the buckling strength and deformations pattern. Kiani et al. [16] used the classical plate theory (CPT) to determine thermal buckling loads of clamped FGM rectangular plates on elastic foundations. Alipour and Shariyat developed semi-analytical solutions for buckling analysis of variable thickness transversely graded [17] and bidirectional FGM [18] viscoelastic circular plates on elastic foundations. Buckling of FGM plates on Pasternak foundations was studied by Thai and Kim [19], using the third-order shear deformation theory. Zhang and Zhou [20] conducted mechanical and thermal post-buckling analyses for FGM rectangular plates resting on nonlinear elastic foundations. Buckling of orthotropic FGM plates surrounded by elastic foundations was treated by Shariyat and Asemi [14], employing the 3D theory of elasticity and B-spline elements. Shariyat and Asemi [21,22] and Asemi and Shariyat [23] studied biaxial and shear post-buckling of the isotropic and auxetic FGM plates, respectively, employing the 3D elasticity theory.

Regarding the skew plates, Ng and Das [24] analyzed buckling of the clamped skew sandwich plates, using Galerkin method. Kamal and

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Durvasula [25] studied free vibration and buckling responses of simply supported skew and trapezoidal plates, using Ritz method. Liao and Lee [26] analyzed stability of the laminated skew plates under biaxial follower forces. Reddy and Palaninathan [27] determined buckling loads of skew laminates subjected to uniaxial and biaxial loadings, employing the CPT and the finite element method (FEM). Utilizing CPT and the shear-deformation first-order theory (FSDT), Wang [28] investigated buckling of the skew laminates, using a Rayleigh-Ritz method. Babu and Kant [29] and Kant and Babu [30] employed finite element models based on the first- and higher-order shear-deformation theories and skew boundary transformation to obtain critical loads of the laminated composite sandwich skew plates. Singha et al. [31] studied buckling and Post-buckling responses of laminated composite skew plates subjected to uniaxial compression and uniform temperature rise, using the FEM. Stability of simply supported isosceles trapezoidal plates subjected to in-plane compression was studied by Mania [32] semi-analytically, based on the CPT and coordinate system transformation. Hu et al. [33] employed a non-linear in-plane shear material constitutive model for FE buckling analysis of the skew laminated composite plates. Ganapathi et al. [34] analyzed buckling of FGM skew plates subjected to uniaxial, biaxial, and shear loadings using the FEM and FSDT. Prakash et al. [35] investigated thermal post-buckling response of the FGM skew plates, using the FEM-based FSDT. Civalek [36–38] presented formulations, geometric transformations, and various numerical approaches for free vibration and buckling of the arbitrary straight-sided quadrilateral isotropic plates. Malekzadeh [39] employed the FEM and DQM to study buckling behavior of the FGM quadrilateral plates. Vosoughi et al. [40] conducted a thermal post-buckling analysis on laminated composite skew plates with temperature-dependent properties, using FE-based FSDT formulations and DQM. Upadhyay and Shukla analyzed buckling and post-buckling behaviors of laminated composite, sandwich [41], and FGM [42] skew plates, using a higher-order shear-deformation theory and a linear geometric mapping.

The foregoing brief literature survey reveals that thermal buckling of a general quadrilateral plate fabricated from an orthotropic or/and auxetic FGM has not been investigated to date. This task is undertaken in the present research. Furthermore, the plate is assumed to be resting on an elastic Winkler-Pasternak elastic medium. Edges of the plate are assumed to be either simply supported or clamped. The defined problem is general enough to cover wide ranges of the practical applications. The stability equations are derived through transformation of the governing equations of the plate from the rectangular Cartesian coordinates to the natural coordinates and discretization of the resulting equations by means of the DQM.

2. Development of the governing equations

2.1. Description of the geometry and material properties

Geometric parameters of the considered general quadrilateral orthotropic auxetic FGM plate are defined in Fig. 1, where θ is the orthotropy angle of the anisotropic functionally graded material with respect to the x axis. Sides of the plate are neither of equal length nor parallel.

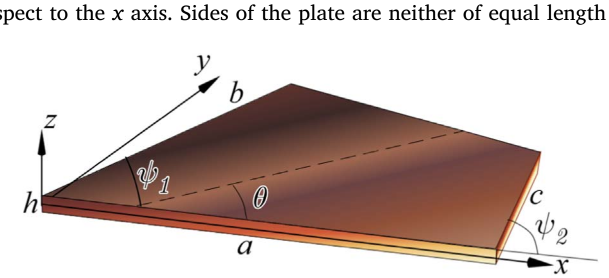

Fig. 1. Geometric parameters of the considered orthotropic functionally graded general quadrilateral plate.

The thermoelastic constitutive law of the plate may be expressed as:

$$\begin{pmatrix}\sigma_x\\ \sigma_y\\ \sigma_{xy}\end{pmatrix}=\begin{pmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16}\\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26}\\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}\end{pmatrix}\begin{pmatrix}\varepsilon_x-\varepsilon_x^T\\ \varepsilon_y-\varepsilon_y^T\\ \varepsilon_{xy}-\varepsilon_{xy}^T\end{pmatrix},\quad \begin{pmatrix}\sigma_{yz}\\ \sigma_{xz}\end{pmatrix}=\begin{pmatrix}\bar{Q}_{44} & \bar{Q}_{45}\\ \bar{Q}_{45} & \bar{Q}_{55}\end{pmatrix}\begin{pmatrix}\gamma_{yz}\\ \gamma_{xz}\end{pmatrix} \tag{1}$$

where ε^T denotes vector of the thermal strains and $\bar{Q}$ is the transformed stiffness matrix of the material. Assuming that the stiffness of the material increases as one proceeds from the mid-layer toward the top and bottom layers (to compensate for the maximum in-plane stresses that occur in these layers) and the Poisson ratio is constant in the transverse direction, the following exponential law may be employed to describe transverse variations of each element of the transformed stiffness matrix [6,14]:

$$\bar{Q}_{ij}=\bar{Q}_{ij}^{ref}\, e^{\left|\frac{pz}{h}\right|},\quad i,j=1,2,4,5,6 \tag{2}$$

$\bar{Q}_{ij}^{ref}$ is magnitude of the stiffness matrix element at the mid-layer of the plate. As Fig. 1 shows, the transverse z coordinate is measured from the mid-layer of the plate. Therefore:

$$E_i=E_i^{ref}\, e^{\left|\frac{pz}{h}\right|},\quad i=1,2\quad;\quad G_{ij}=G_{ij}^{ref}\, e^{\left|\frac{pz}{h}\right|},\quad i,j=1,2,3 \tag{3}$$

The thermal strain vector may be written as:

$$\boldsymbol{\varepsilon}^T=\begin{Bmatrix}\varepsilon_x^T\\ \varepsilon_y^T\\ \varepsilon_{xy}^T\end{Bmatrix}=\Delta T(x,y)\begin{Bmatrix}\alpha_x\\ \alpha_y\\ \alpha_{xy}\end{Bmatrix} \tag{4}$$

where $\Delta T(x,y)$ is the temperature rise with respect to a stress free condition and α_{ij} $(i,j=x,y)$ are the thermal expansion coefficients in the geometric space and may be related to those of the material principal coordinates as:

$$\begin{Bmatrix}\alpha_x\\ \alpha_y\\ \alpha_{xy}\end{Bmatrix}=\begin{pmatrix}\cos^2\theta & \sin^2\theta\\ \sin^2\theta & \cos^2\theta\\ 2\cos\theta\sin\theta & -2\cos\theta\sin\theta\end{pmatrix}\begin{Bmatrix}\alpha_1\\ \alpha_2\end{Bmatrix}e^{\left|\frac{pz}{h}\right|} \tag{5}$$

where α_1 and α_2 are the thermal expansion coefficients in directions parallel and normal to the fibers, respectively.

2.2. Basic kinematic and equilibrium equations of the plate

In-plane and transverse variations of the displacement components may be described according the FSDT as:

$$\begin{cases}u(x,y,z)=u_0(x,y)+z\varphi_x(x,y)\\ v(x,y,z)=v_0(x,y)+z\varphi_y(x,y)\\ w(x,y,z)=w(x,y)\end{cases} \tag{6}$$

where u_0, v_0, and w are displacement components of the mid-layer of the plate, in the x, y, and z directions, respectively, and φ_x and φ_y are rotations of the section in the x-z and y-z planes, respectively. The relevant strain-displacement relations are:

$$\begin{aligned}\varepsilon_x &= u_{0,x}+z\varphi_{x,x}, & \varepsilon_y &= v_{0,y}+z\varphi_{y,y},\\ \varepsilon_{xy} &= \tfrac{1}{2}(u_{0,y}+v_{0,x})+z(\varphi_{x,y}+\varphi_{y,x}), & &\\ \gamma_{xz} &= \varphi_x+w_{,x}, & \gamma_{yz} &= \varphi_y+w_{,y}\end{aligned} \tag{7}$$

where the comma symbol in the subscripts stands for a partial derivative with respect to the indicated coordinate. In addition to the edge supports, the plate may be supported by a Winkler-Pasternak elastic foundation. Therefore, the governing equations of the plate in the framework of the FSDT become [43]: