Thin-Walled Structures 119 (2017) 166–177

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Full length article

Saturated impulse for pulse-loaded rectangular plates with various boundary conditions

Xueyu Bai[a], Ling Zhu[a,b,*], T.X. Yu[a,c]

[a] Key Laboratory of High Performance Ship Technology of Ministry of Education, School of Transportation, Wuhan University of Technology, Wuhan, PR China
[b] Department of Mechanical and Aerospace Engineering, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong, PR China
[c] Collaborative Innovation Centre for Advanced Ship and Deep-sea Exploration (CISSE), PR China

ARTICLE INFO

Keywords:
Saturated impulse
Rectangular plates
Boundary condition
Rigid-plastic analysis
Elastic-plastic analysis

ABSTRACT

Saturated impulse refers to the critical value after which the deflection of the beam or plate under pulse loading will not increase with further applied load. This paper investigates this phenomenon for rectangular plates under rectangular pulse-loading with various symmetric boundary conditions, e.g. two opposite edges of the plate are fully clamped while the other two edges are simply supported or free. For rectangular plates with symmetric boundaries under rectangular pressure pulse, the saturated duration and saturated impulse are merely dictated by the deformation mechanism, not by the boundary conditions. Then the effects of the aspect ratio and the boundary condition on the saturation impulse for elastic-plastic plates are revealed through numerical simulation. Bilinear elastic-plastic rectangular plates, with fully clamped boundaries and simply supported boundaries, subjected to rectangular pulse-loading are taken as typical examples. Subsequently, the saturation phenomenon is summarized from square plates to rectangular plates. The correlation is made between the results obtained from elastic-plastic analysis and those predicted by the rigid-plastic approximation. Finally, based on numerical simulation, theoretically and practically meaningful empirical expressions on the saturation phenomenon are proposed to facilitate engineering designs.

1. Introduction

The saturation phenomenon is a special feature in the dynamic plastic response of structures. When a beam or a plate is subjected to intense transverse dynamic loading, it usually undergoes a large deflection. The load-carrying capacity of the beam/plate is greatly enhanced by the membrane forces induced by large deflection. If the beam/plate is subjected to a rectangular pressure pulse with a sufficiently long duration, only the early part of the pulse contributes to the deflection of the beam/plate. Beyond a critical value, i.e., saturated impulse, the deflection of the beam/plate under the pulse loading will no longer increase with further applied load.

Over past decades much effort has been made to study the dynamic response of rectangular plates (e.g. see Ref. [1–6]), which is one of the most popular load-bearing components in many engineering applications. Jones et al. [7,8] performed experiments on the dynamic plastic behavior of fully clamped rectangular plates using sheet explosive. Jones [9] also developed an approximate theoretical procedure to estimate the permanent deflections of beams and plates with finite deflections. Nurick and Martin et al. [10] performed experiments on the dynamic plastic behavior of fully clamped rectangular plates using sheet explosive and the range of deflection-thickness ratio were obtained. Zhu [11] developed a numerical program to analyses the elastic-plastic response of clamped rectangular plates. The tool using the Variational Finite Difference Method (VFDM) whilst the effects of elasticity, finite transverse deformation and material strain hardening are included. Later, this tool was employed to investigated the dynamic inelastic response of rectangular plates under rigid wedge impact (see Ref. [12,13] with particular reference to minor ship collision (see Ref. [14]). Zhu [15] investigated, both experimentally and numerically, the transient deformation modes of square plates under explosive loading. Yu and Chen [16] presented a theoretical procedure of tracing the large deflection dynamic response of rectangular plates considering plastic dissipation of membrane forces. Their predictions on the final deflection coincide excellently with the experimental data for deflection up to 5–10 times the plate thickness. More recently, Hosseini and Abbas [17] established an empirical relationship that describes the deflection created in a rectangular plate struck by a rigid wedge at the plate center with suffcient initial kinetic energy to produce large inelastic deformations. Liu et al. [18] validated the prediction of the total force-

---

[*] Corresponding author.
E-mail address: ZL79111@hotmail.com (L. Zhu).

http://dx.doi.org/10.1016/j.tws.2017.03.030
Received 7 October 2016; Received in revised form 15 March 2017; Accepted 28 March 2017

Notation

A	parameter in the empirical expression of the dimensionless saturated impulse
A_i	area of the ith deformation block
$2B$	breadth of the rectangular plate
E	Young's modulus
H	plate thickness
$\bar{I}$	dimensionless impulse
l_j	length of the jth plastic hinge
$2L$	side length of square plate
m	number of plastic hinge lines
M_0	fully plastic bending moment per unit length
M_e	elastic bending moment per unit length
n	number of active plastic regions
N_0	plastic membrane force per unit length
p_0	initial peak load of pressure pulse
p_y	static collapse pressure
p_{y0}	$12M_0/B^2$
t	time
t_0	loading duration of rectangular pressure pulse
T	response duration of plate
w	transverse deflection of plate
W_0	deflection at plate center
$1/\beta$	aspect ratio of rectangular plate
γ	a parameter in relation to the boundary condition of rectangular plate
η	pressure ratio p_0/p_y
η_0	pressure ratio p_0/p_{y0}
η_1	pressure ratio p_0/p_{y1}
θ	angular at the outer boundaries of active plastic region
λ	a parameter to characterize the partial restraint at boundaries
μ	mass per unit area of plate
ξ	a parameter related to aspect ratio β
ρ	density of material
σ_y	yield stress
ϕ	angle defined in Fig. 2
ω	a parameter defined by Eq. (29)
$()^{sat}$	saturated value
$()_f$	final value
$()_m$	maximum value

displacement response and the shape of the failure modes for rectangular plates through drop weight impact tests and numerical simulations of small-scale rectangular plates. Tavakoli and Kiakojouri [19] investigated the dynamic response of the stiffened plates considering the effect of stiffener configurations using the general purpose finite element package ABAQUS. Several parameters, such as boundary conditions, mesh dependency and strain rate, had been considered in their study.

As mentioned above, the large inelastic deformation response of thin plates subjected to uniform distributed blast loads has received considerable attention with satisfactory correlation between predictions and experiments. However, the predictions have either considered simply supported plates or assumed fully built-in edge conditions, while most experiments have been performed on plates considered fully clamped at the boundary, with only some experiments performed on plates with fully built-in conditions at the boundary. Many scholars have done much work on the effect of boundary conditions for dynamic response of structures. Wood [2] studied the behavior of a rectangular plate with partially restrained supports, and produced a deformation mode on the ground of the experimental results. Manolakos and Mamalis [20] developed the end-fixity coefficients method to predict the upper- and lower-bounds for rigid-perfectly plastic rectangular plates transversely loaded. Various boundary conditions are applied in the investigation. Thomas and Nurick [21] reported that the large inelastic mid-point deformation response is independent of the boundary fixation conditions. Later, Nurick, Gelman, and Marshall [22] highlighted the significant effects of the boundary conditions for the purpose of predicting tearing based primarily on experimental data of clamped circular plates subjected to uniform loaded air blasts. Cui, Hao and Cheong [23] numerically investigates the dynamic buckling of rectangular plates with clamped–clamped boundaries and simply supported-free boundaries subjected to intermediate-velocity impact loads. Bonorchis and Nurick [24] presented a series of experiments examining the effects of welded boundaries on the localised blast load response of mild steel plates. Villavicencio and Soares [25] investigated the impact response of rectangular and square stiffened plates supported on two opposite edges. A model was produced to simulate the boundary conditions.

Although the saturation phenomenon is of great significance to engineering design of beams and plates, insufficient works have been done on it so far. Zhao, Yu and Fang [26] first illustrated the saturated impulse of a structure undergoing large deflection under a moderate rectangular pressure pulse, and made a convincing explanation to the saturation phenomenon. Non-dimensional saturated impulse of simply supported (or clamped) beam was demonstrated. In a different study, they [27] further extended this concept to circular plates, square plates and cylindrical shells under the similar pulse loading. Since the rigid-plastic idealization was adopted in their modeling, the above-mentioned studies of saturated impulse were only referred to the maximum deflection of beams and plates. Then, Zhu and Yu [28] further developed the concept of the saturated impulse with respect to the maximum deformation as well as the final deformation based on elastic-plastic analysis. A clamped square plate was taken as a typical example. The existence of the saturated impulse for elastic-plastic plates was examined; a "saturated duration" for rectangular loading pulse (i.e., the critical pulse length) was proposed; and a transverse displacement curve was provided based on the elastic-plastic numerical calculations.

The purpose of the present work is to explore the effects of aspect ratios and boundary conditions on the saturation phenomenon of rectangular plates. The concept of saturation impulse thereby is extended to rectangular plates with various boundary conditions. The analytical procedure is described briefly in Section 2 and illustrated in Section 3 for the partially restrained rigid-perfectly plastic rectangular plate. The formulae for the saturated duration, saturated impulse and saturated deflection of rectangular plates with various symmetric boundary conditions are derived through the same theoretical approach is described in Section 4, with details given in Appendix A. A series bilinear elastic-plastic rectangular plates subjected to rectangular pulse-loading are analyzed in Section 5 to demonstrate the effects of aspect ratios and boundary conditions on the saturation phenomenon through numerical simulation. Sections 6 and 7 contain the discussion and conclusions.

2. Theoretical method of analysis

An approximate method based on energy balance was developed in Ref. [9] for arbitrarily shaped flat plates subjected to transverse loads which produce finite displacements. Consideration of the influence of finite-deflections, or geometry changes, would introduce membrane forces and add further complexity. To simplify the analysis, the static plastic collapse mechanism for a plate, which undergoes infinitesimal displacements based on the upper bound theorem of plasticity, is adopted, and then it is further assumed that this collapse mechanism for