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Curvature effects on the eigenproperties of axisymmetric thin shells

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ABSTRACT

A set of linear elastic homogeneous isotropic axisymmetric thin shells of revolution with plane projection of radius R varying with the height c of the pole, to keep constant mass, is introduced. Their curvature and dynamical properties depend on the ratio c/R, and their linear dynamics is investigated by standard modal analysis, adopting a commercial code, and accounting for curvature. Natural frequencies for a given mode are linear with c/R, decrease for membrane modes, and increase for transverse modes. Thus, membrane and transverse modes may shift as curvature grows; graphical and numerical results are reported.

1. Introduction

Thin elements find applications in many fields of engineering, because of their high strength-to-weight ratio (at least under certain external loads), their easy mounting and monitoring. Thin beams, plates and shells are found in civil, industrial, and aerospace structures: rack scaffolds, cranes, multistory buildings, bridges, long-span decks, vaults, motorcycle, automobile, and wing frames, airplane fuselage. As for two-dimensional elements, leaving it aside more general shapes of the middle surface, axisymmetric plates and shells have wide ranges of application, from acoustical instruments and musical elements to cooling towers at power stations, water tanks, industrial chimneys and containment vessels, portions of rockets and missiles. Plates and shells are usually said to be thin when their average thickness is very small compared with the characteristic dimensions of their middle surface. In such a case, the shearing strain between the middle surface and the filaments along their thickness is usually neglected; this theory of linearly elastic, homogeneous, and isotropic thin plates and shells is well known in the literature, and it may be found in both standard and more recent monographs [1–5].

A key point in the investigation of the behaviour of thin elements is the study of their linear dynamics, i.e., of their natural frequencies and free vibration modes for various boundary conditions and shapes. This is fundamental for the investigation of basic mechanical features of the considered elements: linear dynamics, indeed, provides information on the actual stiffness of the member, on its attitude towards buckling and flutter and to resonance under exciting external forces; in addition, in recent times, the monitoring of the dynamic response is a key point for damage detection, hence continuous structural control; moreover, wave transmission (for instance, acoustic) cannot but rely on the information on basic linear dynamics of the considered element. Leaving it aside the huge amount of literature for beams, when dealing with thin shells, in addition to the already quoted [1–5], the well known monographs [6–9] may be cited, where many benchmark cases, as well as a vast amount of quotations to existing papers, are presented. Some recent papers dealing with the dynamics of axisymmetric isotropic shells are, for instance: [10], where experimental and numerical investigation conducted to assess the dynamic behaviour of combined conical vessels is reported; [11], where a numerical approach for the evaluation of the exact dynamic stiffness matrix for each segment of composed (segmented) axisymmetric shells is derived, then assembled and made singular to provide the natural frequencies; [12], where a method for analysing linear and nonlinear vibrations of circular cylindrical shells with different boundary conditions is presented, and comparisons with experiments and finite-element analyses are carried out; [13], which reviews recent research on the strength, stability and vibration behaviour of liquid-containment shell structures. An example of recent investigations on the applications of shell dynamics to sound and acoustics is [14], where a submarine hull is modelled as a cylindrical shell with internal bulkheads and ring stiffeners, and its dynamic response is investigated. No mention will be made here to the numerous publications on anisotropic, composite, non-linear, functionally graded, piezoelectric axisymmetric shells.

The dynamics of thin plates is basically different from that of shells, in that in-plane and transverse vibration are uncoupled in plates, and usually coupled in shells; this is a well known feature of flat and curved elements. Among the vast literature on the subject, the recent paper [15] may be quoted, describing vibration of axisymmetric plates in

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conjunction with a fluid environment, as the basis for acoustic investigations. The review paper [16] may also be quoted for the vibration of axisymmetric shells with and without fluid interaction. In addition, it shall not be forgotten that axisymmetric shells may model musical instruments such as tibetan bells and bowls, cymbals or gongs, investigated in [17,18].

All the preceding quoted literature usually refers to a benchmark scheme, i.e., an element, the geometrical characteristics of which are fixed and parametrized by some quantities, on which the solution of interest in engineering is made to depend. By varying those parameters, one investigates, and has the response, of a wide class of elements, all sharing the same shape. The aim of this contribution is different, in that it wishes to investigate how the material response of an axisymmetric structural element evolves and changes with a remarkable change of the shape of the element itself, once fixed its mass (i.e., the quantity of material employed in the element). Indeed, it is understood that a different mechanical response is expected when, starting with the same amount of material, an axisymmetric plate evolving into an axisymmetric shell is imagined. Of course, this implies that, by the same amount of material, and simply changing the initial shape, one may imagine to tune the structural response of the element in such a way that, e.g., the acoustic pressure field has a desired shape in the neighbourhood of the element, thus opening the way to some idea of shape optimization. The aim of this paper is to investigate such a variation of linear dynamic response, by arranging a family of structural axisymmetric elements sharing the same volume but having variable middle surface, starting from a flat one to a hemispherical one. This implies that the linear response of this family of axisymmetric thin elements will be followed, thus describing how the natural angular frequencies and vibration modes depend on the curvature of the element. This is obtained letting the curvature of the middle surface of the element vary as a function of the ratio c/R of the height c of the pole of the axisymmetric shell to the radius R of its circular plane projection.

The solution for the simplest cases, corresponding to the two extremes of the family (the axisymmetric plate and the hemispherical shell) can be found in closed form, while the rest of the results are obtained numerically by a commercial code. Some graphs illustrating the behaviour of the natural angular frequencies and the relevant natural vibration modes are provided.

The eigenproperties are found by means of a standard modal analysis based on the field equations for thin shells [8,2]; the effect of the curvature is taken into account according to the investigation in [19]. The solution of the relevant modal equations has been performed numerically by the commercial code COMSOL®. The dependence of the eigenfrequencies for a given eigenmode on the ratio c/R will be searched for: in detail, the variation of such dependence for membrane and transverse modes will be investigated. In accord with this, the possibility of shifts of some modes will be studied, and the dependence of the stiffness of the element of the family on the ratio c/R will be searched for. Some graphical and numerical results are reported. These results will be the basis for work in progress in the field of acoustical emission of these elements.

2. Field equations

Axisymmetric elements can be described geometrically by a set of orthogonal curvilinear coordinates α_i, $i = 1, 2, 3$. The pair $\{\alpha_1, \alpha_2\}$ spans the shell middle surface, which is a surface of revolution. The third coordinate α_3 lies along the unit normal $\mathbf{n}$ to the middle surface at any of its points, and spans the thickness h, which is supposed uniform at all points of the middle surface, and thin with respect to the other characteristic lengths of the element. The position vector $\mathbf{R}$ of a point of the shell is referred to a chosen origin in the Euclidean ambient space, and can be expressed also with respect to a Cartesian ortho-normal triad x_i, $i = 1, 2, 3$, in that $\alpha_i = \hat{\alpha}_i(x_j)$, $i, j = 1, 2, 3$. The vector $\mathbf{R}$ is decomposed into the position $\mathbf{r}$ of the projection of the same point on the middle surface, plus a vector along $\mathbf{n}$, and metrics in the shell is obtained

$$\begin{aligned} d\mathbf{R} &= d\mathbf{r} + d(\alpha_3\mathbf{n}) = \sum_{i=1}^{3} \frac{\partial \mathbf{r}}{\partial \alpha_i} d\alpha_i + (d\alpha_3)\mathbf{n} + \alpha_3 d\mathbf{n}, \\ ds^2 &= d\mathbf{R}\cdot d\mathbf{R} = A_1^2\left(1 + \frac{\alpha_3}{R_1}\right) d\alpha_1^{\,2} + A_2^2\left(1 + \frac{\alpha_3}{R_2}\right) d\alpha_2^{\,2} + d\alpha_3^2 \\ &= \sum_{i=1}^{3} g_{ii}(\alpha_j) d\alpha_i^{\,2} \end{aligned} \tag{1}$$

In Eq. (1) $i=1,2$; the A_i are named Lamé's parameters; the R_i are the radii of curvature of the middle surface along the coordinates α_i; the mixed terms g_{ij} vanish since the α_i are locally orthogonal. Metrics orthogonal to the middle surface is Cartesian.

Strain is the variation of metrics between the reference and the present configuration. If this process involves small displacements with respect to the characteristic lengths of the element, strain can be linearized; its components ε_{ij} are, dropping independent variables from notation,

$$(ds')^2 = \sum_{i,j=1}^{3} G_{ij}(\alpha_k) d\alpha_i d\alpha_j \Rightarrow \varepsilon_{ii} = \frac{1}{2}\frac{G_{ii} - g_{ii}}{g_{ii}}, \quad \varepsilon_{ij} = \frac{G_{ij}}{\sqrt{g_{ii}g_{jj}}} \tag{2}$$

In Eq. (2) the ε_{ii} are elongations of material fibres along the tangents to the α_i at the considered point, and the ε_{ij} are shearing strains between formerly orthogonal pairs of the same fibres.

By Love's hypotheses on thin shells, the displacement components U_1, U_2 along α_1, α_2 (membrane displacements) are linear in α_3, while the transverse displacement U_3 does not depend on α_3; in addition, shearing strains between the normal $\mathbf{n}$ and the middle surface are negligible, like Euler-Bernoulli's hypotheses on slender beams. Then, the coefficients of the linear terms in the membrane displacements are the components of the rotation of the transverse fibres of the shell

$$\begin{aligned} U_1(\alpha_1, \alpha_2, \alpha_3) &= u_1(\alpha_1, \alpha_2) + \alpha_3\beta_1(\alpha_1, \alpha_2) \qquad & \varepsilon_{13} = 0 \Rightarrow \beta_1 = \tfrac{u_1}{R_1} - \tfrac{1}{A_1}\tfrac{\partial u_3}{\partial \alpha_1} \\ U_2(\alpha_1, \alpha_2, \alpha_3) &= u_2(\alpha_1, \alpha_2) + \alpha_3\beta_2(\alpha_1, \alpha_2) & \\ U_3(\alpha_1, \alpha_2, \alpha_3) &= u_3(\alpha_1, \alpha_2) & \varepsilon_{23} = 0 \Rightarrow \beta_2 = \tfrac{u_2}{R_2} - \tfrac{1}{A_2}\tfrac{\partial u_3}{\partial \alpha_2} \end{aligned} \tag{3}$$

Replacing Eq. (3) into Eq. (2) yields the non-vanishing components of linearized strain

$$\begin{aligned} \varepsilon_{11} &= \frac{1}{A_1}\frac{\partial}{\partial \alpha_1}\left[u_1 + \alpha_3\left(\frac{u_1}{R_1} - \frac{1}{A_1}\frac{\partial u_3}{\partial \alpha_1}\right)\right] \\ &\quad + \frac{1}{A_1A_2}\frac{\partial A_1}{\partial \alpha_2}\left[u_2 + \alpha_3\left(\frac{u_2}{R_2} - \frac{1}{A_2}\frac{\partial u_3}{\partial \alpha_2}\right)\right] + \frac{u_3}{R_1} \\ \varepsilon_{22} &= \frac{1}{A_2}\frac{\partial}{\partial \alpha_2}\left[u_2 + \alpha_3\left(\frac{u_2}{R_2} - \frac{1}{A_2}\frac{\partial u_3}{\partial \alpha_2}\right)\right] \\ &\quad + \frac{1}{A_1A_2}\frac{\partial A_2}{\partial \alpha_1}\left[u_1 + \alpha_3\left(\frac{u_1}{R_1} - \frac{1}{A_1}\frac{\partial u_3}{\partial \alpha_1}\right)\right] + \frac{u_3}{R_2} \\ \varepsilon_{12} &= \frac{A_2}{A_1}\frac{\partial}{\partial \alpha_1}\left[\frac{u_2 + \alpha_3\left(\frac{u_2}{R_2} - \frac{1}{A_2}\frac{\partial u_3}{\partial \alpha_2}\right)}{A_2}\right] + \frac{A_1}{A_2}\frac{\partial}{\partial \alpha_2}\left[\frac{u_1 + \alpha_3\left(\frac{u_1}{R_1} - \frac{1}{A_1}\frac{\partial u_3}{\partial \alpha_1}\right)}{A_1}\right] \end{aligned} \tag{4}$$

In Eq. (4), the terms independent of α_3 are membrane characteristic strains, while the terms linear in α_3 are the transverse characteristics strains

$$\varepsilon_{ij} = \varepsilon_{ij}^0 + \alpha_3\kappa_{ij} \tag{5}$$

where: ε_{ij}^0 are the elongation and the shearing strain at the projection of the considered point on the middle surface; the κ_{ij} are the curvature increments of the middle surface. Henceforth, the variables α_1, α_2, on which ε_{ij}^0 and κ_{ij} depend, will be dropped from notation.

Introducing Eqs. (4), (5) into the homogeneous isotropic linear elastic constitutive relations yields the stresses in the shell, by which