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Stochastic nonlinear vibration and reliability of orthotropic membrane structure under impact load

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ABSTRACT

Orthotropic membrane structure is widely applied in construction buildings, mechanical engineering, electronic meters, space and aeronautics, etc. During their serving period, membrane structure is prone to vibrate stochastically and seriously under stochastic dynamic loads, which may lead to structural failure. For this purpose, this paper investigates the stochastic dynamic response and reliability analysis of membrane structure under impact load obeying Gaussian distribution. The equation of stochastic motion of membrane structure is established by Von Karman's large deformation theory. The results of stochastic dynamic response are obtained with perturbation method solving the equation. Then, reliability parameters of extreme value of dynamic response are calculated by Moment method based on first-passage probabilities of level crossing. Furthermore, the theoretical model proposed is validated by experimental study using Monte Carlo method. The effects of parameters including impact velocity, pretension force and radius on structural reliability are discussed in addition. The model proposed herein provides some theoretical basis for the stochastic vibration control and dynamic design of orthotropic membrane structure based on reliability theory.

1. Introduction

Orthotropic membrane structure is widely used in various fields such as construction buildings, mechanical engineering, electronic meters, space and aeronautics, etc. Due to their light weight and low stiffness, membrane structure is quite sensitive to impact loads such as rainstorm, hails, drops, etc. In this condition, severe vibration and large deformation is prone to occur, which may lead to structural failure [1]. Moreover, the impact loads imposed on membrane structure during the serving period possess stochastic characteristics, which can cause stochastic dynamic response following some rules of probability [2]. Therefore, it is necessary to study (i) the dynamic response of membrane structures under stochastic impact load and (ii) the structural reliability.

In recent years, the dynamic response problems of membrane structure under impact load have been studied. York et al. [3] extended the material point method (MPM) to discretize membrane structure, and studied the nonlinear vibration problem of discretized membrane structure combining Lagrangian and Eulerian features. Phoenix and Porwal [4] developed an analytical model for 2D membrane impacted transversely by the ballistic impact load. The results can be applied to the design of fibrous materials such as body armor. Malla and Gionet [5] proposed a three-dimensional membrane structure covered with regolith shielding planed to be a lunar habitat. Then, the dynamic response problem of the membrane structure under impact load is investigated. Zheng et al. [6] studied the nonlinear vibration of orthotropic membrane structure under impact load according to principle of virtual displacement, and the results can provide the accurate theory for the measurement of pretension. Liu et al. [7] investigated the undamped nonlinear vibration response of membrane structure subjected to impact load by analytical and numerical methods. The results obtained can provide some theoretical basis for vibration control and dynamic design of membrane structure. Mostofi et al. [8] carried out the theoretical analysis of fully clamped thin plates under impulsive loading. The study considered the effects of both load and material on dynamic response of structure and predicted final deflection accurately. Li et al. [9] studied nonlinear vibration of membrane structure under impact load combining calculus of variation and multiple scales method theoretically and experimentally.

Despite several studies, the research dealt with dynamic response of membrane structure considering stochastic impact load is still limited, to the best of the authors' knowledge. Gosling et al. [10] applied BS EN

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1990:2002 "Eurocode-Basis for Structural Design" to membrane structures and explained the implications of analysis and design. The safety of membrane structures as an explicit function of uncertainty load is predicted through two examples. Khan et al. [11] reported a vibration-based polydimethylsiloxane (PDMS) membrane type electromagnetic energy harvester (EMEH), and studied its nonlinear behavior under stochastic excitations. Zheng et al. [12] considered the stochastic characteristic of impact load and analyzed the stochastic vibration of membrane structure considering the stochastic characteristic of impact load. Some statistical parameter such as mean value, variance, and mean square value were presented.

According to the literature review above, it can be found that the aforementioned researchers mainly focus on the deterministic dynamic response of membrane structure under certain impact load. However, the impact loads imposed on membrane structure in engineering application possess uncertain characteristics, whose values of frequency and amplitude are variable in natural environment [13]. It will cause the actual deformation larger in a probability than the calculated deformation value obtained from deterministic analysis, which may exceed expected structural response and result in tearing of membrane, even structural failure. Thus, it is worth studying the stochastic dynamic response and reliability of membrane structure under uncertain impact load.

This paper studies the stochastic dynamic response and reliability problems of orthotropic circular membrane structure under uncertain impact load. Firstly, the equation of stochastic motion of membrane structure is established by Von Karman's large deformation theory, and solved by perturbation method. Consequently, the analytical solution of statistical parameters is obtained. Next, the reliability problem is further investigated using moment method based on the first-passage probabilities of level crossing. The structural failure probability and reliability index of extreme value of dynamic response are calculated. Then, the stochastic dynamic response and reliability problems are studied using Monte Carlo method experimentally in order to validate theoretical model. Finally, effects of parameters such as impact velocity, pretension force and radius are discussed.

2. Problem formulation

2.1. Theoretical model

In this study, a circular orthotropic membrane with fixed edges is considered as shown in Fig. 1. The basic assumptions of this model are as follows: (1) the membrane material is continuous, homogeneous and linear elastic; (2) the tensile force only exists in membrane; (3) the membrane could only vibrate vertically; (4) the pretension force remains all along in tangent surface as follows; (5) impact load obeys the Gaussian distribution according to nature rules.

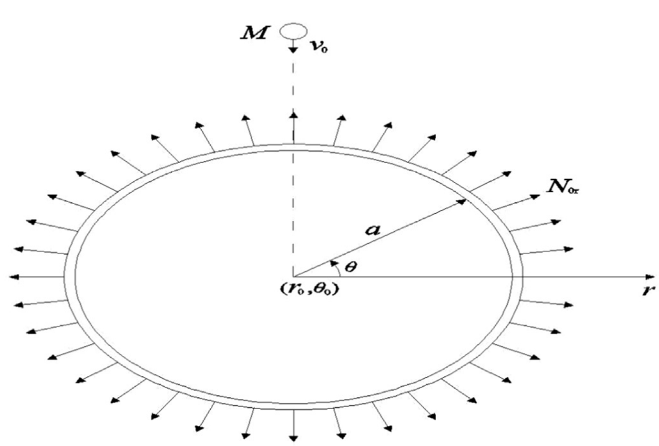

Fig. 1. Orthotropic circular membrane structure subjected to an impact load.

In polar coordinate systems, r and θ denote the parameter of polar radius and polar angle, respectively. The largest polar radius is a. N_{0r} denotes pretension force at edges. The density and thickness of the membrane is ρ and h, respectively. The impact load is a ball that can be considered as a particle. The initial velocity and mass of the ball is v_0 and M respectively. The impact contact point is (r_0, θ_0).

2.2. Impact load and boundary condition

It is assumed that there is only one impact load and the impact load can be defined as the following expression [14].

$$p(r, \theta, t) = F(t)\delta(r - r_0)(\theta - \theta_0), \tag{1}$$

where, $p(r, \theta, t)$ denotes the impact load, $F(t)$ denotes the impact force on the membrane and δ denotes the Dirac function.

There is no deformation for all the points of the membrane before the membrane is impacted. When the load is imposed, the membrane and pellet will start to move as they obtain the velocity. Thus, the initial conditions at the impact position are expressed as

$$\begin{cases} w(r_0, \theta_0, t)|_{t=0} = 0 \\ \frac{\partial w(r_0, \theta_0, t)}{\partial t}|_{t=0} = v_0' \end{cases}, \tag{2a, b}$$

where, w denotes the displacement of membrane and v_0' denotes the velocity of membrane at the moment of load imposing.

Since the actuation duration of the imposed load is very short, the relation between the impact load and the displacement of membrane can be established based on the principle of impact and momentum. The relational expression is given by impulse theory [15] as following

$$\int_0^t F(\tau)d\tau = Mv_0 - M\frac{\partial w(r_0, \theta_0, t)}{\partial t}. \tag{3}$$

After differentiating Eq. (3) with respect to time, it is obtained as follows

$$F(t) = -M\frac{\partial w^2(r_0, \theta_0, t)}{\partial t^2}. \tag{4}$$

The boundary conditions at circumference edge can be expressed as follows

$$w(r, \theta, t)|_{r=a} = 0. \tag{5}$$

2.3. Governing equation of motion

Based on the general theory of plates and shells, the consistency relation between deflection and the additional membrane stress during vibration process can be obtained. If radial displacement at the membrane is assumed to be u_r, the radial and circumferential unit elongations at membrane are ε_r and ε_θ, respectively, and the geometrical equation in large deflection could be expressed by Von Karman's large deformation theory [16] as following

$$\begin{cases} \varepsilon_r = \frac{\partial u_r}{\partial r} + \frac{1}{2}\left(\frac{\partial w}{\partial r}\right)^2 \\ \varepsilon_\theta = \frac{u_r}{r} \end{cases}. \tag{6}$$

After derivation, Eq. (6) can be expressed without u_r is as following:

$$r\frac{d\varepsilon_\theta}{dr} + \varepsilon_\theta - \varepsilon_r + \frac{1}{2}\left(\frac{dw}{dr}\right)^2 = 0. \tag{7}$$

Since membrane material is orthotropic, rectangular coordinate system (X, Y) is proposed in addition so as to express material characteristics. In the rectangular coordinate system, X and Y represent warp direction and weft direction, respectively. The relationship of stress-strain is expressed as