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Rotation-free isogeometric analysis of functionally graded thin plates considering in-plane material inhomogeneity

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ABSTRACT

We present a rotation free isogeometric analysis formulation based on Kirchhoff-Love theory, which aims to address free vibration and buckling behaviors of functionally graded thin plates with in-plane material inhomogeneity. For Kirchhoff-Love thin plate analysis, construction of C^1 conforming finite element approximation is not straightforward, while isogeometric analysis with high-order continuity splines basis functions is ideally suited for Kirchhoff-Love elements. We first explain the formulations and then provide verification of the present method through numerical examples. Studies on convergence and comparison with reference solutions are demonstrated to show the effectiveness and accuracy of the proposed method. Effects on natural frequencies, critical buckling loads and mode shapes originated from the material inhomogeneity and boundary conditions are numerically investigated.

1. Introduction

Functionally graded materials (FGM) are a special type of composites with continuous variation of material properties in spatial directions. As no material interfaces exist in FGM, the interfacial stress concentration phenomenon which may lead to delamination or debonding can be completely avoided. The FGMs thus have wide applications in many engineering areas including aerospace, transducers, energy transform, biomedical engineering, optics [1].

A number of researchers paid attention to the investigation of FGM structures (including beams, plates and shells) with properties graded in the thickness direction [2–9]. It can be seen from the literature that the static, dynamic, buckling and nonlinear response of FGM beams, plates and shells has been studied extensively. For an overview of FGM beam/plate/shells with material inhomogeneity along the thickness direction the reader is referred to the article by Jha and Kant [10]. However, there are only a few investigations on the structural response of FGM structures with material inhomogeneity along in-plane direction or bi-directions (in the thickness and the in-plane direction). Nemat-Alla [11] used the volume fractions and rules of mixtures to investigate the thermal stresses in FGM plates graded in x- and y- directions. Lü et al. [12] proposed a semi-analytical solution based on differential quadrature method for bi-directional FGM beams. It has shown that the bi-directional FGM has higher capability to reduce thermal stresses than that from conventional unidirectional FGM. By employing Levy's type solution, Yu et al. [13] and Liu et al. [14] studied the bending problem of a thin rectangular plate with in-plane stiffness changing through a power form and the fundamental frequency of plates with in-plane material inhomogeneity, respectively. Recently, Amirpour et al. [15] derived an analytical solution for the deflection of functionally graded thick plates with in-plane stiffness variation using higher order shear deformation theory. Even though, analytical solutions can provide benchmark results for assessing approximate theories, only very limited cases can be solved. Therefore, for general cases the numerical methods are necessary.

By employing the meshless local Petrov–Galerkin (MLPG), Qian and Ching [16] studied the static and dynamic behavior of a bi-directional functionally graded cantilever beam. And then, Qian and Batra [17] extended the MLPG to optimize the fundamental frequency of bi-directional functionally graded plates. Goupee and Vel [18] used element free Galerkin method to perform an optimized natural frequency of bi-directional functionally graded beams. Uymaz et al. [19] studied the fundamental frequency of plates with in-plane material inhomogeneity by Ritz method. Xiang et al. [20] investigated the free vibration and the mechanical buckling of plates with in-plane material inhomogeneity using a three dimensional consistent approach based on the scaled

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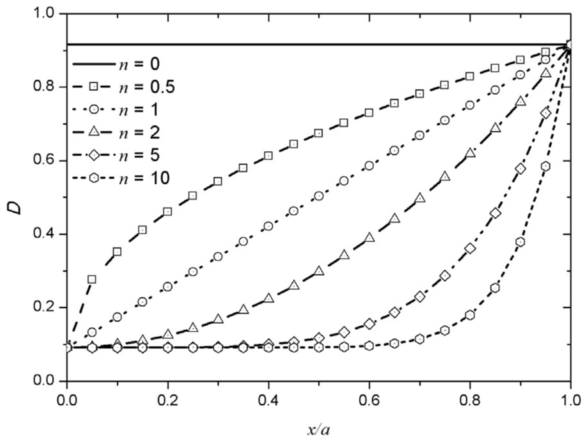

Fig. 1. Variation of rigidity D with different gradient index n.

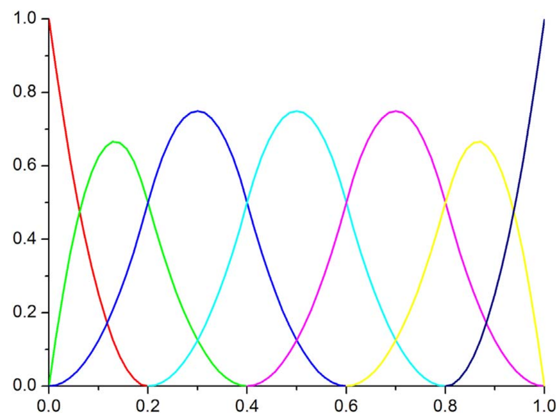

Fig. 2. Quadratic B-spline basis functions.

boundary finite element method. By employing the Hermite radial basis collocation method, Chu et al. [21,22] studied the free vibration and buckling analysis of functionally graded thin plates with in-plane material inhomogeneity.

Considering Kirchhoff thin plate model, it is not trivial to procure C^1 conforming finite element approximation in FEM. In recent years, isogeometric analysis (IGA), e.g., see [23], has attracted considerable attentions for no need of meshing, exact geometry representation, higher-order continuity and simple mesh refinement. In IGA, since the high order continuity NURBS are used as basis functions for analysis, The C^1 continuity requirement of Kirchhoff-Love is easily to be handled without additional efforts. Based on the Kirchhoff-Love theory, Kiendl et al. [24] initially derived a rotation free isogeometric shell formulation, and fully developed for multiple NURBS patches using the bending strip method [25]. Later, the rotation-free isogeometric shell element was extended to large deformation [26], free vibration and buckling analysis of laminated plates [27] and functionally graded plates [28], cloth simulation [29]. Investigated in the previous works, it exhibits that the rotation-free isogeometric plate/shell elements can attain very good accuracy and are efficient for thin plate and shell structures. To the best of the authors' knowledge, no computational approach in terms of the framework of IGA and Kirchhoff-Love has been elaborated to analyze FGM plates with in-plane material inhomogeneity. Thus, in this paper, rotation free isogeometric analysis based on Kirchhoff-Love theory is introduced for the free vibration and buckling analysis of FGM plates with in-plane material inhomogeneity.

This paper is organized as follow. The FGM thin plates with in-plane material inhomogeneity and governing equations based on Kirchhoff-Love are discussed in Section 2. In Section 3, NURBS basis function and rotation free isogeometric plate formulation for free vibration and buckling analysis are presented. Several numerical examples are given in Section 4, followed by concluding remarks in the last section.

Table 1
Normalized frequencies of FGM square plate with parameter γ in different boundary conditions.

(a)SSSS

γ	Method	Number of control points	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
0	IGA	7 × 7	19.7405	49.5295	49.5295	79.1989	101.2744	101.2744
		9 × 9	19.7381	49.3705	49.3705	78.9734	99.1659	99.1659
		13 × 13	19.7377	49.3416	49.3416	78.9356	98.7080	98.7080
	Analytical [21]		19.7392	49.3480	49.3480	78.9568	98.6960	98.6960
	HRBCM [21]		19.6632	49.3620	49.3620	78.9490	98.8862	98.8999
2	IGA	7 × 7	19.9004	49.9071	49.9643	79.6786	101.6802	102.1944
		9 × 9	19.8944	49.7446	49.7642	79.4088	99.5731	99.7647
		13 × 13	19.8932	49.7149	49.7291	79.3636	99.1156	99.2734
	Analytical [21]		19.8948	49.7215	49.7350	79.3844	99.1042	99.2587
	HRBCM [21]		19.9094	49.7151	49.7459	79.3868	99.2417	100.6515

(b)SCSC

γ	Method	Number of control points	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
0	IGA	7 × 7	28.9843	55.0111	70.2747	95.5281	104.9449	133.1751
		9 × 9	28.9555	54.7863	69.4584	94.7184	102.7328	130.5936
		13 × 13	28.9492	54.7383	69.3274	94.5734	102.2343	129.1807
	Analytical [21]		28.9509	54.7431	69.3270	94.5853	102.2160	129.0960
	HRBCM [21]		28.9445	54.8496	69.3869	94.7423	102.4649	129.0501
2	IGA	7 × 7	29.5994	55.5724	71.2765	96.4674	105.4498	135.6784
		9 × 9	29.5292	55.3044	70.2226	95.4153	103.2041	131.5394
		13 × 13	29.5141	55.2460	70.0517	95.2237	102.6962	129.9925
	Analytical [21]		29.5147	55.2498	70.0462	95.2294	102.6770	129.8920
	HRBCM [21]		29.518	55.3641	70.1023	95.3861	103.3145	130.2347