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Investigation on inconsistency of theoretical solution of thermal buckling critical temperature rise for cylindrical shell

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ABSTRACT

In order to solve the inconsistency problem of the theoretical solutions of critical buckling temperature rise for thin cylindrical shells reported in the existing literatures, a first attempt was made in the present study to perform a derivation process on the critical internal force, thermal stress and critical buckling temperature rise for the rectangular thin plate and thin cylindrical shell based on small deformation theory and the Donnell form of the nonlinear equilibrium equations, respectively. Thereafter, the theoretical solutions of the thermal stresses under different boundary conditions and temperature rise variations were determined. The results show that the theoretical solutions of the internal force, thermal stress and critical buckling temperature rise are in good agreement with the numerical results. Finally, the reason leading to the inconsistency of the theoretical solution of the critical buckling temperature rise was elucidated in detail.

1. Introduction

As a general engineering structure, plates and shells have been widely applied into the aerospace, petrochemical, nuclear energy and other industrial fields [1,2]. For ensuring the safety of these structures, it is essential to precisely calculate their strength, stiffness and stability. Recently, it has been arisen many scholars' concern on the thermal buckling behavior of thin plates and shells under thermal environments, such as high temperature storage tank [3,4], conical shell [5], clad steel with coating [6] and railway track [7] etc.

For the thin cylindrical shell subjected to different temperature loads, Ghorbanpour [8] determined the theoretical solution of its critical buckling temperature rise based on the Donnell form of the nonlinear equilibrium equations. Similarly, the theoretical critical buckling temperature rise of the orthotropic laminated cylindrical shell was solved and the influence of length-to-radius ratio on the critical buckling temperature rise was discussed by Eslami and Javaheri [9]. For the composite laminated spherical shell, Darvizeh et al. [10] studied the relation between the number of buckling wave and the thermal strain under different thickness-to-radius ratios and the ply orientation angles under the uniform and the linear temperature load along the wall thickness based on the semi-analytical finite element (FE) method. For the functionally graded material (FGM) plates and shells which are mainly used in aerospace and nuclear energy industries [11–13], Eslami et al. [14], Javaheri and Eslami [15], Wu et al. [16] and Shahsiah et al. [17–19] derived the theoretical solutions of the critical buckling temperature rise of the rectangular plate and the cylindrical shell. In addition, the theoretical solution of the critical buckling temperature rise for the imperfect cylindrical shell was given by Eslami and Shahsiah [20] based on Wan-Donnell model [21].

However, a large amount of the above-mentioned work has mainly focused on the theoretical derivation, but little work has been done on verification of the correctness of the theoretical results. In addition, the present authors found out that there were some differences among different theoretical solutions. For example, the critical buckling temperature rise was written as $\Delta T_{cr} = 0.42h/(\alpha R)$ in the Refs. [8,14,17], it was regarded as $\Delta T_{cr} = 0.61h/(\alpha R)$ in Ref. [16], and $\Delta T_{cr} = 5.3h/(\alpha R)$ in Ref. [22], which will bring some ambiguities for other researchers. Authors [23,24] also demonstrated that the buckling problems are strongly dependent on the constraint condition. However, the current achievements mostly concern the influence of the constraint condition on the buckling behavior of these structures subjected to the mechanical loads, but the study on the thermal buckling problem is not covered so much. In order to clarify the inconsistency problem of the theoretical solution, a first attempt has been made in this work for deriving the theoretical solutions of the critical buckling temperature rise for the rectangular thin plate and thin cylindrical shell based on small deformation theory and the Donnell form of the nonlinear equilibrium equations, respectively. Moreover, the difference of the boundary condition was discussed since they have great effect on the critical

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Nomenclature

E	Young's modulus
L	length of the shell
M	internal moment per unit length
N	internal force per unit length
P	inverse value of N
R	radius of the shell
ΔT	environmental temperature rise
ΔT_{cr}	critical buckling temperature rise
a	length of the plate
b	width of the plate
h	thickness of the plate or shell
m_p	half wave number of plate in x direction
m_s	axial half wave number of the shell
n_p	half wave number of the plate in y direction
n_s	circumferential full wave number of the shell
u	axial deflection of the shell
v	circumferential deflection of the shell
w	normal deflection of plate or radial deflections of the shell
z	radial variable of the shell
α	coefficient of thermal expansion
β	rotational angle of the shell
γ	shear strain
ε	normal strain
κ	curvatures of the shell
μ	Poisson's ratio
σ	normal stress
τ	shear stress
A, B, C, k, λ	constant coefficients

Subscript

m	middle surface of the shell
p	plate
s	shell
x	X (length) direction of the plate or axial direction of shell
y	Y (width) direction of the plate
θ	circumferential direction of the shell
0	initial state critical equilibrium state of stability
1	instability state (buckling state) or the direction of the plate and shell
2	the direction of the plate and shell

value of the buckling behavior. In addition, other work has been focused on verifying the correctness of the internal force and the thermal stress as well as the critical buckling temperature rise based on the FE method.

2. Theoretical derivation of internal force, thermal stress and critical temperature rise

Generally, the theoretical derivation of mechanical behavior of the thin shell is based on a thin plate. However, it can lead to some errors due to the difference between the shell and the plate. For clarifying the inconsistency problem of the critical buckling temperature rise in existing literatures [8,14,16,17,22], the theoretical derivation process of the thin plate and shell are presented in next sections.

In addition, the procedure to obtain the critical buckling temperature rise is executed in the following. The critical internal force was firstly derived according to the equilibrium equations, and the thermal stress and internal force were then obtained based on the constitutive equations. Finally, the critical buckling temperature rise was solved by integrating the internal force, thermal stress and buckling waveform. Thus, the internal force, thermal stress, and critical buckling temperature rise were derived one by one. Note that the influence of the temperature on the material properties was ignored in this work by adopting the same assumption reported in existing literatures [14,16], which indicated that Young's modulus (E), the coefficient of thermal expansion (α) and the Poisson's ratio (μ) were considered as constants.

2.1. Theoretical solution of the critical internal force

2.1.1. Rectangular plate

For the uniform thick rectangular thin plate with four edges simply supported, the equilibrium equation [21] is given as

$$D(w_{,xxxx} + 2w_{,xxyy} + w_{,yyyy}) = N_{x0}w_{,xx} + N_{y0}w_{,yy} + 2N_{xy0}w_{,xy} \tag{1}$$

where $D = Eh^3/[12(1\text{-}\mu^2)]$, N_{x0}, N_{y0} and N_{xy0} are the internal forces along the x, y, xy directions, respectively. And (,) indicates a partial derivative.

If the temperature gradient only distributes along the thickness direction of the plate, thus $N_{xy0} = 0$. Meanwhile, to set $N_{x0} = -P_x$, $N_{y0} = -kP_x$, where P_x is the inverse value of N_{x0}, and k is equal to the ratio between N_{x0} and N_{y0}. Therefore, the equilibrium equation Eq. (1) becomes

$$D(w_{,xxxx} + 2w_{,xxyy} + w_{,yyyy}) = -P_x(w_{,xx} + kw_{,yy}) \tag{2}$$

Defining the expression of the deflection w as

$$w = \sum_{m_p=1}^{\infty}\sum_{n_p=1}^{\infty} A_{m_pn_p}\sin\frac{m_p\pi x}{a}\sin\frac{n_p\pi y}{b} \tag{3}$$

where $A_{m_pn_p}$ is the coefficient to be determined. m_p and n_p denote the half wave numbers in the x, y directions at the occurrence of the thermal buckling, respectively.

Substituting Eq. (3) into Eq. (2) and with simplification

$$\sum_{m_p=1}^{\infty}\sum_{n_p=1}^{\infty} A_{m_pn_p}\left[D\left(\frac{m_p^2}{a^2}+\frac{n_p^2}{b^2}\right)^2 - \frac{P_x}{\pi^2}\left(\frac{m_p^2}{a^2}+k\frac{n_p^2}{b^2}\right)\right]\sin\frac{m_p\pi x}{a}\sin\frac{n_p\pi y}{b} = 0 \tag{4}$$

Eq. (4) is always established before the thermal buckling occurs. Once the buckling takes place, the critical internal force of the plate is obtained in the following

$$P_x = \frac{\pi^2 D}{a^2}\frac{(m_p^2 + n_p^2a^2/b^2)^2}{m_p^2 + kn_p^2a^2/b^2} \tag{5}$$

2.1.2. Cylindrical shell

For the cylindrical thin shell with a uniform wall thickness, the normal strains in the x (axial) and θ (circumferential) directions and the shear strain in the $x\theta$ direction are [17]

$$\begin{cases}\varepsilon_x = \varepsilon_{xm} + z\kappa_x\\ \varepsilon_\theta = \varepsilon_{\theta m} + z\kappa_\theta\\ \gamma_{x\theta} = \gamma_{x\theta m} + 2z\kappa_{x\theta}\end{cases} \tag{6}$$

where z denotes the radial variable with the origin point at the middle surface of the shell, the coordinate system of the thin shell is shown in Fig. 1.

Based on the assumption of the Sanders [21], the relations between the deflections and the strains on the middle surface of the thin cylindrical shell are