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Vibrations of a composite shell of hemiellisoidal-cylindrical shell having variable thickness with and without a top opening

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ABSTRACT

Natural frequencies and mode shapes of a composite shell of revolution consisting of a circular cylindrical shell and a deep or shallow hemi-ellipsoidal shell with variable thickness having a circular cylindrical hole or not are determined by the Ritz method using a mathematically three-dimensional analysis instead of two-dimensional thin shell theories or higher order thick shell theories. The present analysis is based upon the circular cylindrical coordinates while in the traditional shell analyses three-dimensional shell coordinates have been commonly used. Using the Ritz method, the Legendre polynomials, which are mathematically orthonormal, are used as admissible functions instead of ordinary simple algebraic polynomials. Natural frequencies are presented for different boundary conditions. Convergence to four-digit exactitude is demonstrated for the first five frequencies of the composite shells. The frequencies from the present three-dimensional method are compared with those from three types of two-dimensional thin shell theories (finite element method, finite difference method, and numerical integration) by previous researchers. The present analysis is applicable to very thick shells as well as thin shells; and to shallow shells as well as deep shells.

1. Introduction

The joined shells of revolution have many applications in various branches of engineering such as mechanical, aeronautical, marine, civil and power engineering, and so forth. The research on their mechanical behavior such as vibration characteristics under various external excitations and boundary restrictions has great importance in engineering practice [1]. Unlike the case of a single shell, the investigation on free vibration for such composite shell structures is rather limited, owing to the mathematical complexity of shell equations and the difficulty to meet conditions of the connection between two substructures [2]. Lately, static and dynamic behaviors of joined conical-circular cylindrical shells have been studied by some researchers [1,3–7].

Joined hemi-ellipsoidal or spherical and circular cylindrical shells of revolution have been applied to missiles, airplanes, pressure vessels, architecture, submersibles, and many structures in the petro-chemical and nuclear industries. However, very little work can be found on the vibrations of the joined shells of hemi-spherical and circular cylindrical shells [8–11] and the joined shells of hemi-ellipsoidal and circular cylindrical shells [10]. The first investigation of the joined hemi-spherical and circular cylindrical shells was made by Hammel [9]. He obtained an exact result using a series solution. Lee et al. [11] investigated the free vibration characteristics of a joined hemi-spherical and circular cylindrical shell with various boundary conditions. The Flügge shell theory and Rayleigh's energy method were applied. The natural frequencies and mode shapes were calculated numerically and compared with those of the FEM and modal test. In their formulation, the hemi-spherical shell had free boundary condition and the cylindrical shell has simply supported boundary conditions at the join part. Particularly, at the joint, the spherical shell was free to translate along all direction, while the cylinder was restrained to translate along the radial direction. In their modal test, the considered boundary conditions of the joined structure were free-free, simply supported-free, and clamped-free. Free-free and clamped-free boundary conditions in the experiment were obtained by using the wire and welding along the circumferential direction, respectively. The natural frequencies of vibration of cylindrical shells clamped at one end and closed at the other end by different types of shells of revolution (cones, hemi-spheres, hemi-spheroids, etc.) were determined using programs written for the digital computer by Galletly and Mistry [10]. The principal numerical methods of investigation were variational finite-differences and finite elements. Some results obtained by numerical integration of the differential equation of motion using

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Nomenclature

a	length of semi-major axis of the mid-surface of a hemi-ellipsoidal shell radius of the mid-surface of a circular cylindrical shell
A_{ij}, B_{kl}, C_{mn}	arbitrary coefficients
b	length of semi-minor axis of the mid-surface of a hemi-ellipsoidal shell
D_A	Hilbert space
DET	size of determinant
E	Young's modulus
h	shell thickness in the axial direction (z) at $r = 0$ in the case of $R_i = 0$.
h^*,	$\equiv h/H$
H	uniform thickness of a circular cylindrical shell
H^*	$\equiv H/a$
i, j, k, l, m, n	indices for double summation (non-negative integer)
I_V,I_T	defined by Eqs. (21) and (22), respectively
I,J,K,L,M,N	highest degrees of the Legendre polynomial terms
k'	aspect ratio of a spheroid ($\equiv b/a$)
K	stiffness matrix
$\mathrm{K}_{\alpha\beta\hat{\alpha}\hat{\beta}}$,$\mathrm{M}_{\alpha\beta\hat{\alpha}\hat{\beta}}$	submatrix of K and M
($\alpha = i, k, m$; $\beta = j, l, n$: $\hat{\alpha} = \hat{i}, \hat{k}, \hat{m}$; $\hat{\beta} = \hat{j}, \hat{l}, \hat{n}$)	
L	height of a circular cylindrical shell
L^*	nondimensional length of a circular cylindrical shell ($L^* \equiv L/a$)
M	mass matrix
n	circumferential wave number ($n = 0,1,2,\ldots$)
P_n	Legendre polynomial ($n = 0,1,2,\ldots$)
$P_{\alpha\beta}$	$\equiv P_\alpha(\psi)P_\beta(\zeta)$ ($\alpha = i, k, m$, $\beta = j, l, n$)
R_i	radius of an axially circular hole
R_i^*	$\equiv R_i/a$
r	radial coordinate
r,θ, z	circular cylindrical coordinate system
s	mode number
t	time
T	kinetic energy
$T_{\max}$	maximum kinetic energy
TR	total number of the Legendre polynomial terms used in r or ψ direction
TZ	total number of the Legendre polynomial terms used in z or ζ direction
u_r, u_z, u_θ	displacements in the directions of r, z, θ, respectively
U_r,U_z,U_θ	displacement functions of ψ and ζ
V	strain energy
$V_{\max}$	maximum strain energy
x	vector of unknown coefficients
z	axial coordinate
$z_{i,o}$	coordinates of the inner and outer hemi-ellipsoidal surfaces for $r \geq 0$, respectively
α	arbitrary phase angle
Γ_1, Γ_2	constants, defined by Eq. (24)
δ_{ij}	Kronecker delta
ε	$\equiv \varepsilon_{rr} + \varepsilon_{zz} + \varepsilon_{\theta\theta}$
ε_{ij}	tensorial strain
ζ	non-dimensional axial coordinate ($\equiv z/L$)
$\zeta_{i,o}$	$\equiv z_{i,o}/L$
$\eta_{r,\theta,z}$	functions of ψ and ζ depending upon the geometric boundary conditions
θ	circumferential coordinate
κ_i	functions defined by Eq. (23) ($i = 1,2,\ldots,6$)
λ, G	Lamé parameters
Λ	domain of a joined hemi-ellipsoidal and circular cylindrical shell
ν	Poisson's ratio
ρ	mass density per unit volume
σ_{ij}	tensorial stress
ψ	non-dimensional radial coordinate ($\equiv r/a$)
ψ, θ, ζ	non-dimensional circular cylindrical coordinates
ω	natural frequency
Ω	square of non-dimensional frequency ($\equiv \omega^2 a^2 \rho/G$)
0^{A}	circumferential wave number for axisymmetric modes
0^{T}	circumferential wave number for torsional modes
2DS	2-D thin shell theory
3DR	3-D Ritz method
•	time derivative
,	spatial derivative
$\langle f, g\rangle$	$\equiv \eta(\psi, \zeta) \iint_\Lambda f(\psi, \zeta)\; g(\psi, \zeta)\;\; \psi\, d\zeta d\psi$

Runge-Kutta techniques and by series solutions were also given for purposes of comparison. They utilized the kinematic relations due to Novozhilov, Flügge, or Reissner. Recently, Morsbøl et al. [12] studied the elastic waveguide properties of an infinite pipe with circular cross section whose radius varies slowly along its length. The equations governing the elastodynamics of such shells are derived analytically, approximated asymptotically in the limit of slow axial variation, and solved by means of the WKB-method (Wentzel-Kramers-Brillouin). However, these studies were all based upon *thin* shell theories, which are mathematically two-dimensional (*2-D*). That is, for thin shells one assumes the Kirchhoff hypothesis that normals to the shell middle surface remain normal to it during deformations (vibratory, in this case), and unstretched in length. Conventional shell theory is only applicable to *thin* shells. A higher order thick shell theory could be used which considers the effects of shear deformation and rotary inertia, and would be useful for the *low* frequency modes of *moderately thick* shells. Such a theory would also be *2-D*. Also all the literatures mentioned above were confined to *uniform* shell thickness. However, recently, some researchers [13,14] investigated the free vibrations of laminated composite doubly-curved shells, singly-curved shells and plates with *variable thickness* by the Generalized Differential Quadrature (GDQ) method.

Shell analysis makes simple kinematic assumptions about the variation of displacements through the shell thickness. Almost all such shell analyses assume, at most, constant plus linear variation. Such assumptions reduce the 3-D theory to a 2-D one, characterized by the middle surface displacement. And such analyses are typically accurate for thin or moderately thick shells of homogeneous and isotropic materials, and a least for the lower frequencies (i.e., long wave lengths of mode shapes). But for thicker shells or higher frequencies, a 3-D analysis becomes necessary for accurate frequencies. The present analysis uses the 3-D equations of the theory of elasticity. The 3-D equations are only limited to small strains. No other constraints are placed upon the displacements. This is in stark contrast with the classical 2-D thin shell theories and 2-D thick shell theories, which make very limiting assumptions about the displacement variation through the shell thickness. However, Beresin et al. [15] applied approximate asymptotic theories to the total synthesis of the dispersion curves for a cylindrical shell as for a 3-D elastic body. The Kirchhoff-Love theory and the theory of high frequency long wave vibrations were used in the vicinity of zero frequency and in the vicinities of the thickness resonance frequencies, respectively. The theory of high frequency short wave vibrations was used outside these vicinities. It was shown that the range of applicability of the theory of high frequency, short wave vibrations overlaps those of the Kirchhoff-Love theory and the theory of high frequency, long wave vibrations. Mekhtiev and Bergman [16] investigated the forced vibrations of a transverse isotropic hollow cylinder, loaded by an axisymmetric harmonic force at the butt ends using the homogeneous