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# Thermal buckling of thin spherical shells under uniform external pressure and non-linear temperature

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ABSTRACT

The thermal buckling of the shells is among the popular topics in solid mechanics. To date, most studies adopt the linear constitutive equation method. However, the non-linear temperature is important when studying thermal buckling. In this study, the non-linear constitutive equation of isotropic materials is derived using the tensor method to obtain the stability equation of axisymmetric spherical shells. Moreover, quadratic non-linear constitutive equations are applied to study the thermal buckling of spherical shells, and the heat stability equations of spherical shells expressed by displacements are obtained. Considering the common function of external pressure and temperature, the potential energy function of the spherical shell expressed in displacement is obtained using the principle of least potential energy. Moreover, the Ritz method is used to study the thermal buckling of simple, supported shells. The changing trend of the critical pressure caused by the temperature change of the thin spherical shells is analyzed, as well as the influence of the temperature nonlinearity on the critical pressure.

## 1. Introduction

In recent years, the thermal buckling of plates and shells has increasingly attracted attention. Several references exist on the thermal buckling of cylindrical shells. In 1973, Gupta and Wang [1] studied thermal buckling of orthotropic circular cylindrical shells under uniform temperature fields; they considered the expansion coefficient to obtain the critical temperature using the Ritz method. In 1989, based on the energy method, Wilcox [2] found the Donnell shell theory and obtained the critical temperature of simple supported cylindrical shells, using the Galerkin method of potential energy principle. During 1996–2001, Eslami [3–5] studied buckling and thermal buckling of elastic, composite, and imperfect cylindrical shells. In 2002, Wang [6] analyzed non-linear thermal buckling of laminated composite cylindrical shells with local delamination. In 2003, Shahsiah [7] studied the buckling analysis of functionally graded cylindrical shells subjected to different types of thermal loads in simple, supported boundary conditions, and Shahsiah and Eslami [8] analyzed the thermal buckling of the functionally graded cylindrical shells using the classical and improved mixed stability equations. In 2005, Wu [9] discussed the thermal buckling of cylindrical shells made of functionally graded materials (FGM) based on the Donnell shell theory, as well as the equilibrium and stability equations of cylindrical shells under thermal loads. In 2008, Shariyat [10] studied the temperature dependence of the dynamic thermal buckling of functionally graded composite cylindrical shells under combined axial and external pressures. In 2012, Hui-shen [11] analyzed thermal and posted buckling of carbon nanotube-reinforced composite cylindrical shells with FGM. From 2014–2015, Duc [12,13] studied non-linear buckling of imperfect and eccentrically stiffened metal-ceramic-metal S-FGM and thin circular cylindrical shells with temperature-dependent properties in thermal environments.

Conical shells have attracted the attention of scholars. In 2006, Bhangale [14] studied linear thermoelastic buckling and free vibration behavior of functionally graded, truncated conical shells. In 2007, Sofiyev [15] analyzed the thermal elastic stability of functionally graded, truncated conical shells. In 2008, based on the Sander non-linear shell theory, Naj [16] studied thermal and mechanical instabilities of functionally graded, truncated conical shells. In 2013, Torabi [17] analyzed the linear thermal buckling of truncated hybrid FGM conical shells. The stability equation was established using the minimum potential energy criterion, and the critical buckling temperature difference was obtained using the Galerkin method. In 2015, Sofiyev

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[18–20] analyzed the thermal buckling of functionally graded conical shells in depth, and Seidi [21] studied temperature-dependent buckling analysis of the sandwich-truncated conical shells using FG facesheets.

Moreover, scholars were concerned about the thermal buckling of spherical shells. In 2001, Eslami [22] studied the thermal buckling of thin spherical shells. In 2006, Shahsiah and Eslami [23] described the thermal instability of functionally graded spherical, shallow shells based on the Mushtari-Vlasov-Donnell theory, and Shahsiah [24] studied the stability of thermal buckling of the isotropic, shallow spherical shells under three different temperature loads. In 2010, Eslami [25] studied the thermal buckling of FGM and obtained the critical temperature of the shallow spherical shells under uniform external pressure, temperature, external pressure, and temperature. In 2010, Xu [26] studied the non-linear stability of double-deck reticulated circular shallow spherical shells. In 2011, Mao [27] studied the non-linear dynamic response for the functionally graded shallow spherical shells under low-velocity impact in a thermal environment. In 2012, Sabzikar Boroujerdy [28] studied the thermal buckling of functionally graded shallow spherical shells and the critical temperature of shallow spherical shells with uniform temperature and three kinds of temperature loads along the radial uniform variation and directions. From 2014–2016, Anh and Duc [29–32] studied the non-linear stability of the annular and shallow spherical shells. The elastic foundation of the FGM under external pressure and temperature was analyzed.

Thermal buckling is among the popular issues in solid mechanics. The non-linear temperature is important in studying thermal buckling. In this study, the non-linear constitutive equation of isotropic materials was derived using the tensor method, and the stability equation of axisymmetric spherical shells was obtained. The quadratic non-linear constitutive equations were applied in studying the thermal buckling of spherical shells, and the heat stability equations of spherical shells expressed by displacements were obtained. Considering the common function of external pressure and temperature through the principle of least potential energy, the potential energy function of the spherical shell expressed in displacement was obtained, and the Ritz method was used to study the thermal buckling of simple, supported shells. The changing trend of the critical pressure caused by the temperature change of the thin spherical shells was analyzed, and the influence of the temperature nonlinearity on the critical pressure is analyzed.

## 2. Basic equation

Defect-free, thin spherical shells are studied, as shown in Figs. 1 and 2. The displacements of the normal direction along the middle plane and radial direction are $w$ and $\theta$, respectively.

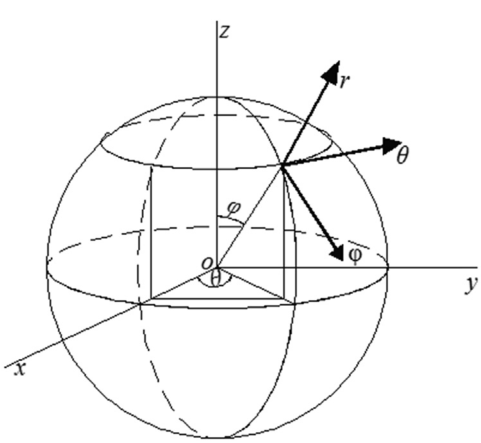


**Fig. 1.** Spherical coordinate system.

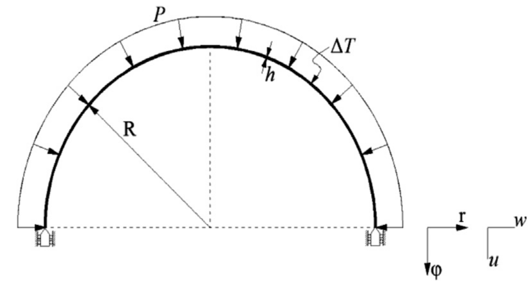


**Fig. 2.** Load diagram of thin spherical shell.

### 2.1. Geometric equations

Due to the axial symmetry in the shell, for each point, the two displacements along the meridian direction and the normal direction are defined as $u$ and $w$, whereas $v = 0$. In the spherical coordinate system, the radius of the curvature, $R_1 = R_2$, and the Lame coefficients of spherical shells are:

$$H_1 = R,\quad H_2 = R\sin\varphi,\quad H_3 = 1$$

The strain in the spherical shells can be realized at any point in the spherical coordinate system [37]. With definition $\varphi\varphi = \varphi,\quad \theta\theta = \theta$:

$$\begin{cases} \varepsilon_\varphi = \frac{1}{R}(u^* + w),\ \varepsilon_\theta = \frac{1}{R}(u\cot\varphi + w) \\ \chi_\varphi = \frac{1}{R^2}(w^* - u)^*,\ \chi_\theta = \frac{\cot\varphi}{R^2}(w^* - u) \end{cases} \tag{2.1.1}$$

is obtained,

where $\frac{\partial(\quad)}{\partial\varphi} = (\quad)^*$. The following equations use the same tag.

### 2.2. Thermal constitutive equation of spherical shells

Considering only the cases of the initial value and increment of temperature, the constitutive equation of the non-linear thermal stress of isotropic materials was studied [36] (Figs. 2–4).

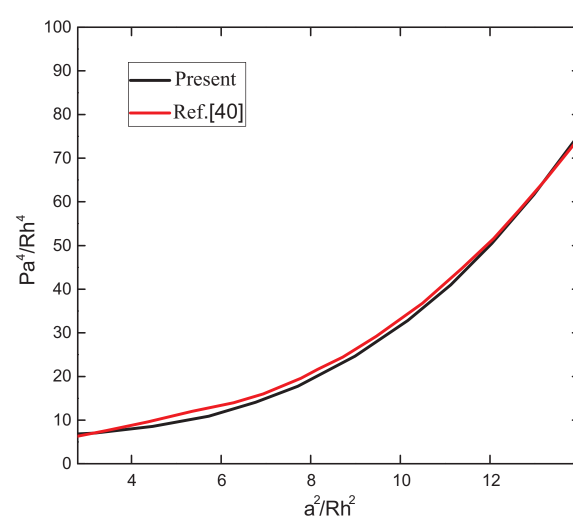


**Fig. 3.** Comparison of dimensionless upper buckling loads for isotropic thin shallow spherical shells.

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