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# Large-amplitude vibration of sigmoid functionally graded thin plates with porosities

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A R T I C L E I N F O

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A B S T R A C T

This research focuses on the large-amplitude vibration of sigmoid functionally graded material (S-FGM) thin plates with porosities. Porosities in S-FGM plates can happen due to technical issues during the preparation of S-FGMs. Two types of porosity distribution, i.e., even and uneven distribution, are taken into account. The material properties of S-FGM plates with porosities change smoothly along the thickness direction based on the sigmoid distribution law, which is described by modified piecewise functions. The geometrical nonlinearity is considered by applying the von Kármán non-linear plate theory. The nonlinear governing equation of S-FGM plates with porosities is derived using the D′Alembert's principle. By applying the Galerkin method with the first three modes, the governing equation is discretized to three ordinary differential equations. Then, the method of harmonic balance is used to solve these discretized equations. Analytical results are verified numerically with the adaptive step-size fourth-order Runge-Kutta method. The stability of the steady-state response is examined by means of the perturbation technique. Furthermore, the maximum amplitudes of each mode during the vibration period are obtained and shown in the neighborhood of the fundamental mode. Study demonstrates that the S-FGM plates with porosities possess hardening spring characteristics in nonlinear frequency response. Moreover, a complex multi-solution phenomenon occurs in the present dynamic system which is rooted from the nonlinear mode interaction. Finally, investigation is made on the effects of porosity along with other key parameters on large-amplitude vibration response of porous S-FGM plates.

## 1. Introduction

Functionally gradient materials (FGMs) are a new member of advanced composite materials that possesses microscopical heterogeneity. Compared with traditional composites, FGMs have many advantages such as smooth stress distribution, less stress concentration and high joint strength of different materials. Therefore, FGMs have been applied widely in aerospace, medicine, power machinery, chemistry and electronics. FGMs can be synthesized using the preparation techniques including self-propagating high temperature synthesis method, multi-step sequential infiltration method and non-pressure sintering method.

When fabricating FGMs, however, porosities or micro-voids may appear inside the materials because of the technical issues. For instance, Zhu et al. [1] found many porosities can appear inside the materials when fabricating FGMs via the non-pressure sintering method. The existence of porosities weakens significantly the strength of FGMs. Additionally, it was detected that [2] porosities occur mostly in the middle region of the FGMs in the preparation process by using the multi-step sequential infiltration method. This phenomenon is due to the fact that it is hard to infiltrate the secondary material into the middle area perfectly, whereas it is easier to infiltrate the material into the top and bottom areas, resulting in less porosities in these two zones. In view of the existence of porosities inside FGMs, it is necessary to take into account porosity influence on vibration behavior of FGM structures containing porosities.

FGM plates are the fundamental element in many engineering structures and thus have attracted considerable research interests [3]. While most studies of FGM plates focused on stress and buckling analyses [4], vibration analyses on FGM plates are not large. Among them, Kolakowski and Mania [5] studied the dynamic response of a FGM plate under in-plane pulse force. Adopting the classical thin plate theory, Malekzadeh and Monajjemzadeh [6] analyzed the nonlinear response of a FGM plate subjected to a moving force. By using the finite element method, Gupta et al. [7] investigated natural frequencies of a shear deformable FGM plate with various boundary conditions. With the consideration of hygrothermal and elastic foundation effects, Sobhy [8] studied the free vibration of a FGM sandwich plate utilizing a four-variable shear deformation plate theory. Jędrysiak [9] calculated the

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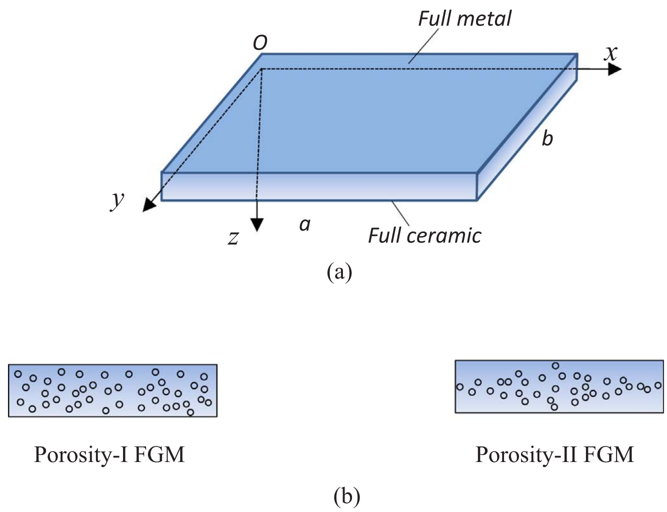


**Fig. 1.** A rectangular S-FGM plate with porosities: (a) Cartesian coordinate system; (b) plate cross-sections for two types of porosity.

frequencies characteristics of FGM plates carrying a micro-structure. Alijani et al. [10] utilized pseudo-arc-length continuation techniques and Lagrange method to study non-linear dynamic responses of FGM plates. Alibeigloo and Alizadeh [11] analyzed the static and free vibration characteristics of FGM sandwich plates; two types of conformation of the plate were considered in the study. Adopting von Kármán non-linear plate theory, Allahverdizadeh et al. [12] used single-mode approximation to obtain the dynamic responses of a FGM plate. Based on the Mindlin plate theory along with modified couple stress theory, Ke et al. [13] carried out the axisymmetric non-linear free vibration analysis on FGM microplates. Using the third-order shear deformation theory, Zhang et al. [14] studied the chaotic motion of simply-supported thick FGM plates through the method of multiple scales. By utilizing the two-dimensional and quasi three-dimensional shear deformation theories, Akavci and Tanrikulu [15] calculated the natural frequencies of FGM plates. Non-linear vibrations of FGM plates under different edge constrains were analyzed by Hao et al. [16,17], who concentrated on the periodic, quasi periodic and chaotic vibrations of the composite structures. Zhang et al. [18] showed that the external force can be used as a controller to tune the nonlinear response of FGM plates from periodic vibration to chaotic vibration. Recently, utilizing von Karman-type nonlinear relations, Wang and Zu [19] investigated the nonlinear steady-state responses of rectangular FGM plates which have longitudinal speed and couple with fluid. In the foregoing literature, power-law function or exponent function were adopted to define the constituent volume variation of FGMs.

While perfect FGMs have been studied widely on their vibration behaviors, only a few researches are reported on vibration of FGMs with porosities. Wattanasakulpong and Ungbhakorn [20] calculated the linear and nonlinear frequencies of FGM Euler–Bernoulli beam with porosities. Wattanasakulpong and Chaikittiratana [21] also studied the free vibration of a Timoshenko beam with porosities, where the unevenly distributed porosities were taken into account. Considering thermal effect, Ebrahimi et al. [22] analyzed natural frequencies of FGM Euler–Bernoulli beams with porosities via the differential transform method. Ait Atmane et al. [23] calculated the natural frequencies of a porous FGM beam resting on elastic foundations. Considering a porous FGM nanoplate resting on Winkler–Pasternak foundations, porosity effect on the free vibration of the system was shown by Mechab et al. [24]. In a word, these papers all concentrate on *free* vibration of FGMs with porosities.

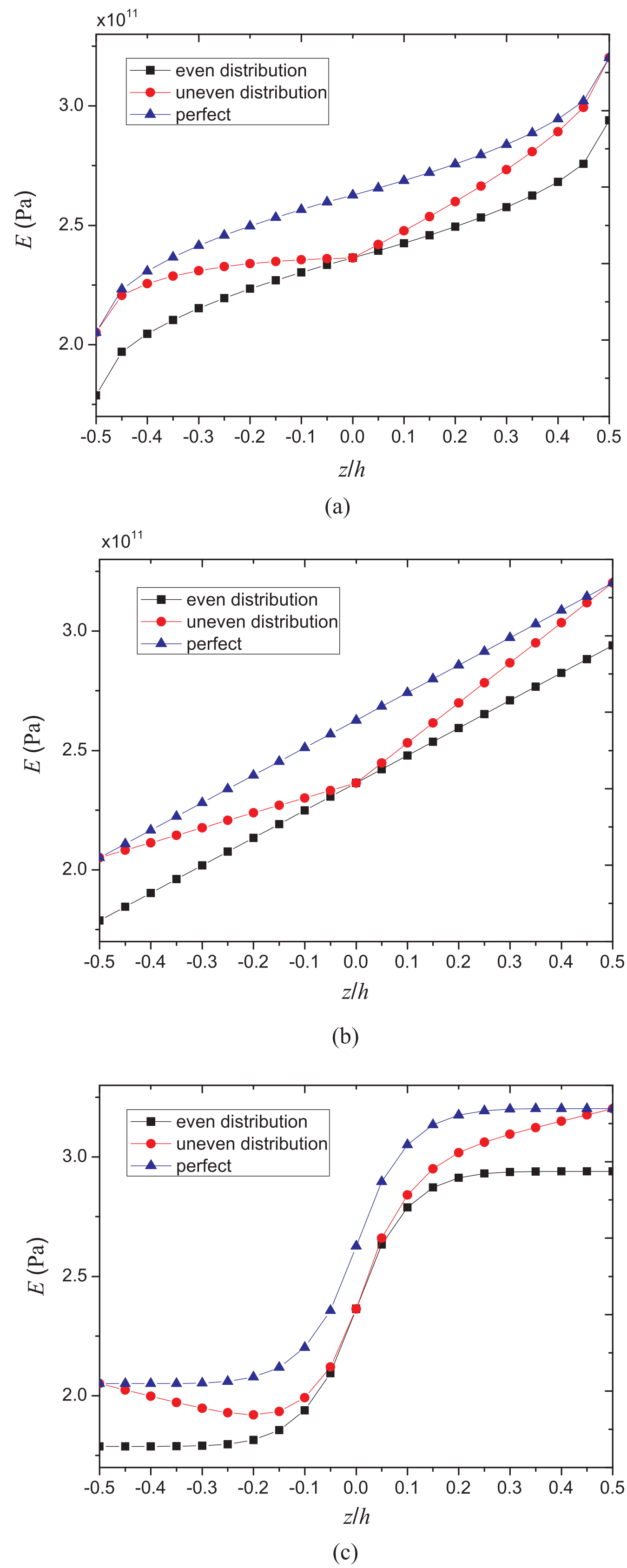


**Fig. 2.** Variations of Young's moduli in perfect and porous S-FGM plates with $\alpha = 0.1$: (a) $N = 0.5$; (b) $N = 1$; (c) $N = 6$.