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Flexural-torsional buckling of general cold-formed steel columns with unequal unbraced lengths

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ABSTRACT

The design of cold-formed steel columns must consider flexural buckling, torsional buckling, and flexural-torsional buckling. The American Iron and Steel Institute incorporated equations for the critical elastic buckling loads corresponding to these failure modes in the North American Specification for the Design of Cold-Formed Steel Members. These equations were originally developed for columns with consistent boundary conditions for all three modes. However it is common in practice to have different unbraced lengths for major axis flexure, minor axis flexure, and torsion. Furthermore, it is common for certain members to be oriented such that intermediate bracing restraint directions do not align with the principal axes. This paper investigates and develops a general formulation of the column buckling equation to incorporate unequal unbraced lengths and non-principal axes.

1. Introduction

Cold-formed steel structural members are often used in framing configurations where intermediate bracing provides a reduced unbraced length for one direction and twisting. A common example is a Z purlin as shown in Fig. 1. Since a Z shape is point-symmetric with the shear center coinciding with the centroid, there is no interaction between torsional buckling and flexural buckling. Therefore the buckling limit is simply the smaller of the two buckling loads.

The flexural buckling load is normally calculated using the conventional $P=\pi^2 EI/L^2$, where I is the minor principal axis moment of inertia. But where the bracing directions do not coincide with the principal axes, the impact of having different unbraced lengths is not evident. The coupling of the two flexural modes requires further investigation.

Similarly, a singly-symmetric C shape is commonly used with intermediate bracing to reduce the unbraced length for minor axis buckling and torsional buckling. Since the shear center for a C shape does not coincide with the centroid, interaction between flexural buckling and torsional buckling occurs. The unbraced lengths for flexure and torsion can be different, therefore complicating the interaction between them.

The column buckling equations used in cold-formed steel design today were investigated by Timoshenko et al. [2] among others. They were further studied by Chajes et al. [3] for development of the design criteria in the AISI Specification [1]. These buckling equations were developed using principal axes and equal unbraced lengths for all modes. This paper expands on their excellent work to consider the more general case of unequal unbraced lengths and non-principal axes. Numerous symbols are used in this investigation which are defined Section 7.

2. Development

2.1. General case

The development of the critical buckling load for a general cold-formed steel shape must consider a combination of flexural buckling and torsional buckling. Fig. 2 represents an arbitrary non-symmetric cross section oriented to centroidal x and y axes which represent the two orthogonal directions of translational bracing. These axes need not be the principal axes.

The application of axial load P at the centroid C with sufficient magnitude will produce buckling where the cross-section displaces u and v in the x and y directions, and rotates about its shear center by angle ϕ. The centroid translates from C to C_1, and the shear center translates from O to O_1. The rotation causes the centroid to move to its final position C_2.

To maintain equilibrium, the displaced cross-section develops moments about the x and y axes, which are the product of the axial load P and the x and y displacements from C to C_2, as shown in Eqs. (1) and (2).

$$M_x = -P(v - x_o\phi) \quad (1)$$

$$M_y = -P(u + y_o\phi) \quad (2)$$

The stiffness relationship between moment and deflection for non-principal axes must consider unsymmetrical bending. The general form of this relationship is a pair of differential Eqs. (3) and (4) as developed by Timoshenko [2] and others, which involves the product of inertia I_{xy}.

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Nomenclature

A	Area of cross-section
C_w	Torsional warping constant
E	Modulus of elasticity
G	Shear modulus of elasticity
J	Saint-Venant torsion constant
I_x, I_y	Moments of inertia about x and y axes
I_{xy}	Product of inertia about x and y axes
I_2	Moment of inertia about minor principal axis
K_f	Effective length factor for coupled flexural buckling
K_x, K_y, K_t	Effective length factors for buckling about x axis, y axis, and torsion
L	Column length
L_f	Half-wavelength for coupled flexural buckling (smaller of L_x and L_y)
L_x, L_y, L_t	Unbraced lengths for buckling about x axis, y axis, and torsion
M_x, M_y	Moments about x and y axes
P	Critical elastic buckling axial load
P_x, P_y, P_t	Critical axial loads for elastic buckling about x axis, y axis, and torsion
P_{fx}, P_{fy}	Coupled critical axial loads for flexural buckling about the x and y axes
P_{fxy}	Coupled critical axial load component attributed to unsymmetrical bending
r_o	Polar radius of gyration about shear center
r_x, r_y	Radius of gyration about x and y axes
r_2	Radius of gyration about minor principal axis
T_z	Torsion per unit length of column
u, v, ϕ	Buckling displacements in the x and y directions, and angle of twist
u'', v'', ϕ''	Second derivative of buckling displacements with respect to z
u'''', v'''', ϕ''''	Fourth derivative of buckling displacements with respect to z
x, y	Orthogonal coordinate axes of cross-section corresponding to bracing directions
x_o, y_o	Coordinates of shear center relative to centroid of cross-section
z	Longitudinal axis of column
α, β, γ, δ	Dimensionless factors used in polynomial form of buckling equation
σ	Critical elastic buckling axial stress
σ_{ex}, σ_{ey}, σ_t	Critical axial stress for elastic buckling about x axis, y axis, and torsion
σ_{fx}, σ_{fy}	Coupled critical axial stress for flexural buckling about the x and y axes
σ_{fxy}	Coupled critical axial stress component attributed to unsymmetrical bending

Equating these to the moments defined by the buckling equilibrium relationships in Eqs. (1) and (2) provides two differential equations with three unknowns: u, v, and ϕ.

$$M_x = EI_x v'' + EI_{xy} u'' = -P(v - x_o\phi) \quad (3)$$

$$M_y = EI_y u'' + EI_{xy} v'' = -P(u + y_o\phi) \quad (4)$$

A third relationship is required involving torsion, which was investigated by Timoshenko [2]. Similar to the flexure equations, the stiffness relationship is equated to the buckling equilibrium relationship, both in terms of the torsion per unit length T_z, as shown in Eq. (5). Although this torsion development was presented using principal x and y axes, no assumptions were made that required principal axes. This relationship is applicable to any section orientation.

$$T_z = EC_w\phi'''' - GJ\phi'' = P(x_o v'' - y_o u'') - Pr_o^2\phi'' \quad (5)$$

The solution to these three simultaneous differential equations is developed here using a pinned end column of length L, and subsequently generalized for other cases. Thus we will assign the following boundary conditions: $u = v = \phi = 0$ and $u'' = v'' = \phi'' = 0$, at $z = 0$ and $z = L$.

The solution for u, v, and ϕ are therefore in the forms shown in Eq. (6). The first buckling mode corresponds to one half-wavelength, where $n_1 = n_2 = n_3 = 1$. To accommodate different unbraced lengths, greater values of n may be used to produce unbraced lengths of L/n, and the nodes where the displacements are zero would correspond to the brace points.

$$u = A_1 \sin\frac{n_1\pi z}{L} \quad v = A_2 \sin\frac{n_2\pi z}{L} \quad \phi = A_3 \sin\frac{n_3\pi z}{L} \quad (6)$$

As illustrated in Fig. 3, we will let $L_y = L/n_1$, $L_x = L/n_2$, and $L_t = L/n_3$. It should be noted that any set of unbraced lengths can be accommodated mathematically by defining an imaginary column whose length is a common multiple of the three unbraced lengths, or $L = LCM(L_x, L_y, L_t)$.

For the general case using non-principal axes, the flexural modes are coupled such that they have the same half-wavelength and buckling occurs about a non-orthogonal axis. The bracing directions do not align with the buckling direction, but only a small component of a translational restraint vector is required to create an inflection point. Therefore the unbraced flexural span is the distance between brace points, regardless of bracing direction.

For the purpose of this investigation, the coupled flexural mode solution is assumed to have a consistent half-wavelength throughout the column. This requires L_x to be a multiple of L_y or vice-versa, and therefore the half-wavelength is the smaller of L_x and L_y. Defining L_f as the flexural half-wavelength, the displacement functions and their derivatives are then defined as follows:

$$u = A_1 \sin\frac{\pi z}{L_f} \quad v = A_2 \sin\frac{\pi z}{L_f} \quad \phi = A_3 \sin\frac{\pi z}{L_t} \quad (7)$$

$$u'' = -A_1\frac{\pi^2}{L_f^2}\sin\frac{\pi z}{L_f} \quad v'' = -A_2\frac{\pi^2}{L_f^2}\sin\frac{\pi z}{L_f} \quad \phi'' = -A_3\frac{\pi^2}{L_t^2}\sin\frac{\pi z}{L_t} \quad \phi'''' = A_3\frac{\pi^4}{L_t^4}\sin\frac{\pi z}{L_t} \quad (8)$$

Substituting these forms into the three differential Eqs. (3), (4), and (5) produces the following set of simultaneous equations:

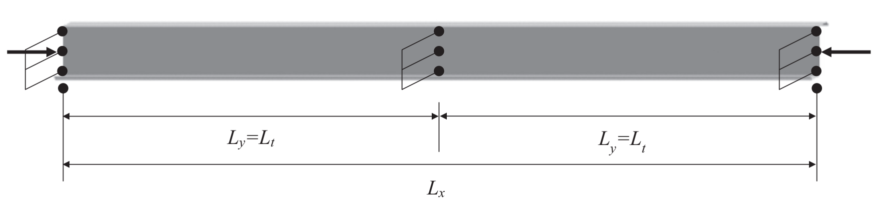

Fig. 1. Typical bracing configuration where unbraced lengths are different.