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Analyses of thin-walled sections under localised loading for general end boundary conditions – Part 2: Buckling

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ABSTRACT

Thin-walled sections under localised loading may lead to buckling of the sections. This paper briefly introduces the development of the Finite Strip Method (FSM) for buckling analyses of thin-walled sections under localised loading for general end boundary conditions. This method is benchmarked against the Finite Element Method (FEM).

For different support and applied loading conditions, different functions are required for flexural and membrane displacements. In Part 1- Pre-buckling described in the companion paper, the analysis provides the computation of the stresses for use in the buckling analyses in this paper. Numerical examples of buckling analyses of thin-walled sections under localised loading with different end boundary conditions are also given in the paper in comparison with the FEM.

1. Introduction

Thin-walled plates and sections subjected to localised loading and experiencing plate buckling have been studied over a long period by numerous investigators who mainly focused on web plates of sections under concentrated load. Two comprehensive investigations in this research area were Khan and Walker [1] for the buckling of plates under localised loading and Johansson and Lagerqvist [2] for the resistance of plate edges under localised loading. In the application of the Generalized Beam Theory (GBT), Natário et al. [3] further extended investigations for beams under concentrated loading. The results for plates, unlipped channel sections and I sections from the GBT have been benchmarked against previous research and the Shell Finite Element method (SFE).

The Finite Strip Method (FSM) developed by Cheung [4] is an efficient method of analysis in comparison with the FEM. This method is used extensively in the Direct Strength Method (DSM) of design of cold-formed sections in the North American Specification for the Design of Cold-Formed Steel Structural Members AISI S100-2012 [5] and the Australian/New Zealand Standard AS/NZS 4600:2005 [6]. It is therefore essential to extend the FSM of buckling analysis to localised loading. The FSM was applied in Chu et al. [7] and Bui [8] to the buckling analysis of thin-walled sections under more general loading conditions, where multiple series terms were used to capture the modulation of the buckles. The limitation of these investigations is that the transverse compression and shear are not included. Hancock and Pham [9] applied the FSM to the buckling analysis of thin-walled sections subjected to shear forces. More recently, Hancock and Pham [10] have extended the FSM to the analysis of thin - walled sections under localised loading for simply supported boundary condition using multiple series terms. In the longitudinal direction, a pre-buckling analysis was performed to compute stresses prior to the buckling analysis using these stresses. Solution convergence with increasing number of series terms was provided. However, in practice, cold-formed members are connected together by welds or bolts so that the end boundary conditions are expected to be different from simply supported. Thus, it is necessary to extend this method to the analysis of thin-walled sections under localised loading for general end boundary conditions.

In this Part 2 – Buckling, the paper briefly introduces the functions used to compute the stress distributions in the strips of the structural member for different end boundary conditions as described in the companion paper Part 1-Pre-buckling. In addition, the theory of the FSM for buckling analysis of thin walled sections under localised loading for general end boundary conditions is developed. Numerical examples have been performed by the FSM built into the THIN-WALL-2 program developed by the authors [11]. The numerical solutions are compared with those from the analyses by the Finite Element Method

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(FEM) on ABAQUS [12] to validate the accuracy including a convergence study.

2. Strip buckling displacements

2.1. Flexural buckling displacement

An isometric view of flexural displacements of a strip is shown in Fig. 3 of the companion paper Part 1 – Pre-buckling.

The flexural deformations w of a strip can be described by the summation over μ series terms as:

$$w = \sum_{m=1}^{\mu} f_{1m}(y) X_{1m}(x) \tag{1}$$

where:

μ is the number of series terms of the harmonic longitudinal function,

$X_{1m}(x)$ is the function for longitudinal variation, as described in Part 1 - Pre-buckling

$f_{1m}(y)$ is a polynomial function for transverse variation. This function for the m^{th} series term is given by:

$$f_{1m}(y) = \alpha_{1Fm} + \alpha_{2Fm}\left(\frac{y}{b}\right) + \alpha_{3Fm}\left(\frac{y}{b}\right)^2 + \alpha_{4Fm}\left(\frac{y}{b}\right)^3 \tag{2}$$

$\{\alpha_{Fm}\}$ is the vector polynomial coefficients for the m^{th} series term which depend on the nodal line flexural deformations of the strip,

$$\{\alpha_{Fm}\} = [\alpha_{1Fm} \quad \alpha_{2Fm} \quad \alpha_{3Fm} \quad \alpha_{4Fm}]^T \tag{3}$$

t, b and L are the strip thickness, width and length respectively.

The flexural deformations w can be written in matrix format:

$$w = \sum_{m=1}^{\mu} X_{1m}(x)[\Gamma_{FL}][C_F]^{-1}\{\delta_{Fm}\} \tag{4}$$

where:

$$\{\alpha_{Fm}\} = [C_F]^{-1}\{\delta_{Fm}\}$$

$$f_{1m}(y) = [\Gamma_{FL}]\{\alpha_{Fm}\}$$

$$[\Gamma_{FL}] = [1 \quad (y/b) \quad (y/b)^2 \quad (y/b)^3]$$

$\{\delta_{Fm}\}$ is the flexural displacement vector for nodal line displacements for the m^{th} series term

$[C_F]$ is the evaluation matrix of the flexural displacement functions at the nodal lines, given in Appendix A.

In the computation of the flexural potential energy described later, the derivatives of the flexural deformation are required. The derivatives used are as follows:

$$\frac{\partial w}{\partial x} = \sum_{m=1}^{\mu} X'_{1m}(x)[\Gamma_{FL}]\{\alpha_{Fm}\} \tag{5}$$

$$\frac{\partial w}{\partial y} = \sum_{m=1}^{\mu} X_{1m}(x)\frac{1}{b}[\Gamma_{FT}]\{\alpha_{Fm}\} \tag{6}$$

where $[\Gamma_{FT}] = [0 \quad 1 \quad 2(y/b) \quad 3(y/b)^2]$

2.2. Membrane buckling displacement

An isometric view of membrane displacements of a strip is shown in Fig. 4 of the companion paper Part 1 – Pre-buckling.

The membrane deformations in the longitudinal and transverse directions of a strip can be described by the summation over μ series terms as:

$$v = \sum_{m=1}^{\mu} f_{vm}(y) X_{1m}(x) \tag{7}$$

$$u = \sum_{m=1}^{\mu} f_{um}(y) X_{2m}(x) \tag{8}$$

where:

$X_{1m}(x)$and$X_{2m}(x)$ are the longitudinal variation functions for the membrane transverse v and longitudinal u deformations respectively, as described in Part 1 – Pre-buckling

$f_{vm}(y)$and$f_{um}(y)$ are the transverse variations. These functions for the mth series term are given by:

$$f_{vm}(y) = \alpha_{1Mm} + \alpha_{2Mm}\left(\frac{y}{b}\right) \tag{9}$$

$$f_{um}(y) = \alpha_{3Mm} + \alpha_{4Mm}\left(\frac{y}{b}\right) \tag{10}$$

$\{\alpha_{Mm}\}$ is the vector of polynomial coefficients for the m^{th} series term which depend on the nodal line membrane deformations of the strips

$$\{\alpha_{Mm}\} = [\alpha_{1Mm} \quad \alpha_{2Mm} \quad \alpha_{3Mm} \quad \alpha_{4Mm}]^T \tag{11}$$

The membrane deformations of the strip can be written in matrix format:

$$v = \sum_{m=1}^{\mu} X_{1m}(x)[\Gamma_{Mv}][C_M]^{-1}\{\delta_{Mm}\} \tag{12}$$

$$u = \sum_{m=1}^{\mu} X_{2m}(x)[\Gamma_{Mu}][C_M]^{-1}\{\delta_{Mm}\} \tag{13}$$

where:

$$\{\alpha_{Mm}\} = [C_M]^{-1}\{\delta_{Mm}\}$$

$$f_{vm}(y) = [\Gamma_{Mv}]\{\alpha_{Mm}\} \quad \text{and} \quad f_{um}(y) = [\Gamma_{Mu}]\{\alpha_{Mm}\}$$

$$[\Gamma_{Mv}] = [1 \quad (y/b) \quad 0 \quad 0] \quad \text{and} \quad [\Gamma_{Mu}] = [0 \quad 0 \quad 1 \quad (y/b)]$$

$\{\delta_{Mm}\}$ is the vector of membrane displacement for the m^{th} series term

$[C_M]$ is the evaluation matrix of the membrane displacement functions at the nodal lines, given in Appendix A.

In the computation of the membrane potential energy described later, the derivatives of the membrane deformations are required. The derivatives used are as follows:

$$\frac{\partial v}{\partial x} = \sum_{m=1}^{\mu} X'_{1m}(x)[\Gamma_{Mv}]\{\alpha_{Mm}\} \tag{14}$$

$$\frac{\partial u}{\partial x} = \sum_{m=1}^{\mu} X'_{2m}(x)[\Gamma_{Mu}]\{\alpha_{Mm}\} \tag{15}$$

3. Membrane stresses

3.1. Membrane stress calculation

The membrane stresses of a strip for the kth series term of the pre-buckling analysis as in Part 1 – Pre-buckling are given by:

$$\{\sigma_{Mk}\} = [D_M]\{\in_{Mk}\} \tag{16}$$

where$[D_M]$ is the property matrix of membrane displacement, given in Appendix A

$\{\in_{Mk}\}$ is the membrane strain vector:

$$\{\in_{Mk}\} = [B_{Mk}]\{\alpha_{Mk}\} \tag{17}$$

Hence:

$$\{\sigma_{Mk}\} = [D_M][B_{Mk}]\{\alpha_{Mk}\} \tag{18}$$