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Chaotic bubbles and phase locking for a shaker system in the vicinity of three coexisting critical points

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ABSTRACT

In this paper, the dynamical model of the shaker system, Poincaré maps, Jacobian matrix and power spectrum are established. Different phase-locking phenomena and chaotic bubbles are investigated in the vicinity of three coexisting critical points including Hopf–Hopf bifurcation point, 1:3 resonance point and 1:4 resonance point. In two strong resonance cases, phase-locking dynamics and associated bifurcations are easily to occur. Coexisting attractors have also been introduced to provide mechanisms for chaotic bubbles with connections between pieces. The occurrence of phase locking on doubling torus to multi-period leads to interruption of torus-doubling bifurcation. Isolated chaotic bubbles are birth via period-doubling bifurcation of such a multi-period. Phase-locking phenomena on T^2 torus are also observed in such a neighborhood of critical points. The number of periods on torus by phase locking can be identified by power spectrum methods. The system parameters may be optimized by studying of phase-locking dynamics of this system.

1. Introduction

In the last decade, chaos and bifurcation have become a focal point for non-linear problems in the scientific community [1–9]. Most mechanical dynamical systems are non-linear in nature, and can be described by the non-linear equations of motion. A shaker system is often encountered in practice and it is used to screening sort by applying vibro-impact mechanics. Repeated impacts usually occur whenever the components of a vibrating system collide with rigid obstacles or with each other. The principle of operation of shakers is based on the impact action for moving bodies. Researches into the vibro-impact problems have important significance in optimization design of machinery with rigid obstacles or clearances, noise suppression and reliability analyses, etc. Because of the existence of impacts, the dynamics of the system is discontinuous and strongly non-linear. It can exhibit rich and complicated dynamical behaviors and it is also a good testing benchmark for non-linear theories [10–20]. On the one hand, impacts between the components take great disadvantage to the system, so the collision should be tried to avoid when the optimization design of machinery with gaps is considered. In the past several years, many important problems on vibro-impact dynamics have been studied. Shaw and Homles [10], Ivanov [11], Nordmark [12], Whiston [13] and Peterka [14] studied the singularities of maps derived from vibro-impact systems. The problems of global bifurcations have received great attention in Refs. [15,16]. The classical pattern of period-doubling bifurcation cascade was observed numerically by Thompson [17]. Iooss [18] studied the subharmonic bifurcation in resonant cases for

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maps. Recently, a few researchers began to focus attention on the phenomena of Hopf bifurcations of vibro-impact systems and studied quasi-periodic or chaotic motion [21–25]. Xie [21] and Wen [22] investigated codimension two (cod-2) bifurcations corresponding to double eigenvalue-1. Dynamics of a two-degree-of-freedom (2-dof) vibro-impact system in resonance is considered by Ding and Xie [23]. Period-doubling bifurcations and routes to chaos are analyzed in 2-dof vibratory system in [24]. Two types of codimension-3 bifurcations and non-typical routes to chaos of a shaker system are investigated by Zhang et al. [25]. Such critical points and coexisting attractors play a central role to study dynamical transitions [26–30]. However, there are few studies on dynamics in some neighborhood of coexisting critical points. In particular, phase-locking phenomena and different chaotic bubbles have not been reported in such cases.

The dynamical model for a shaker system, Poincaré sections and power spectrum are established in the paper. The purpose of the present study is to focus attention on phase-locking dynamics and chaotic bubbles in some neighborhood of multiple coexisting critical points including Hopf–Hopf bifurcation point, 1:3 resonance point and 1:4 resonance points. Chaotic bubbles with connections between pieces in Poincaré sections can be observed, which is called chaotic rings. An interesting feature is interruption of torus-doubling bifurcation and appearance of weakly chaotic rings. We show phase locking (frequency locking) dynamics and existence of coexisting attractors play a central role for such dynamical transitions. Multistability has also been introduced to provide mechanisms for chaotic rings. Isolated chaotic bubbles are birth via period-doubling bifurcation of such a multi-period. Phase-locking phenomena on T^2 torus are also observed in such a neighborhood of critical points. The paper is the continuation of many investigations and explains in more detail interesting and new dynamical behaviors near the multiple coexisting bifurcation points, which develop the results obtained by other models.

The organization of this paper is as follows: in Section 2, we present the mechanical model of a shaker, Poincaré map and power spectrum. In Section 3, we obtained the Jacobian matrix in details. Phase-locking dynamics and chaotic bubbles are investigated by studying numerical simulations in Section 4. Finally concluding remark is given in Section 5.

2. The mechanical model, Poincaré map and power spectrum

The mechanical model for a shaker system is shown in Fig. 1. It is a representative model for the vibro-impact system. The principle of operation of shakers is based on the impact action for moving bodies. This type of system is often encountered in practice, for instance in the models of hammer-like devices. The masses M_1 and M_2 are connected to linear springs with stiffness K_1 and K_2, and linear viscous dashpots with damping constants C_1 and C_2. The excitation on mass M_1 is harmonic with amplitudes P. The excitation frequency Ω and the phase τ are the same for both masses. The mass M_3 impacts against M_1 when M_3 sinks from the upward side of M_1 due to gravitation. The impact is described by a coefficient of restitution R, and it is assumed that the duration of impact is negligible compared to the period of the force. Damping in the mechanical model is assumed as proportional damping of the Rayleigh type.

Between impacts, the governing equations are written in a non-dimensional form

$$\begin{bmatrix} 1 & 0 \\ 0 & \alpha \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} 2\zeta(1+\beta) & -2\zeta\beta \\ -2\zeta\beta & 2\zeta\beta \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} 1+\gamma & -\gamma \\ -\gamma & \gamma \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \sin(\omega t + \tau) \tag{1}$$

$$\ddot{x}_3 = -\delta \tag{2}$$

And the impact equation of mass M_1 and M_3 is

$$\dot{x}_{1-} + \mu\dot{x}_{3-} = \dot{x}_{1+} + \mu\dot{x}_{3+}, \quad \dot{x}_{1+} - \dot{x}_{3+} = -R(\dot{x}_{1-} - \dot{x}_{3-}) \tag{3}$$

where $\dot{x}_{1-}$ and $\dot{x}_{3-}$ represent respectively velocities of mass M_1 and M_3 immediately before impact. And where, $\dot{x}_{1+}$ and $\dot{x}_{3+}$ represent respectively velocities of mass M_1 and M_3 immediately after impact.

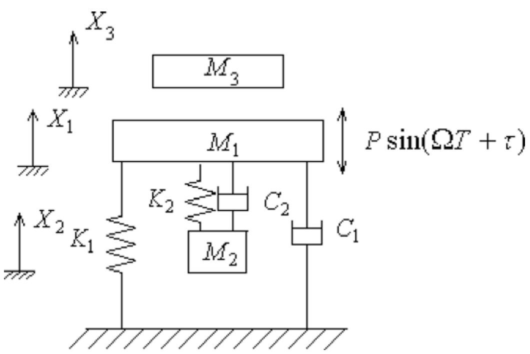

Fig. 1. The model of a shaker system.