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Nonlinear bending, postbuckling and snap-through of circular size-dependent functionally graded piezoelectric plates

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ABSTRACT

In the present work, the nonlinear thermoelectrical stability of perfect/imperfect circular size-dependent functionally graded piezoelectric plates is studied according to modified couple stress theory. The second, concurrent aim is to address snap-through phenomenon in the thermally preloaded plates due to concentrated/uniform lateral loads. Ritz finite element method is implemented into virtual displacement principle to construct the matrix representation of nonlinear governing equations. Under certain circumstances, bifurcational instability may occur in which case a direct displacement control scheme is utilized. In other cases, the response is unique and stable to which any standard load control strategy seems appropriate and thus Newton-Raphson method is selected. Standard load control strategies, however, fail to trace nonlinear equilibrium paths through limit points and path following methods must be employed in snap-through problems. Being more popular among the existing path following solution methods, cylindrical arc-length method is adopted. Two types of thermal loading as well as two cases of boundary conditions are considered. Moreover, various parametric studies are conducted to assess the influence of involved parameters.

1. Introduction

Piezoelectric materials (PMs), standing as a novel class of materials, are widely used due to their distinct properties so as to postpone buckling or control shape [1,2]. The advantages of structures made of PMs have necessitated more investigations on their behavior. Having found a wide range of applications, functionally graded materials (FGMs) are a new class of microscopically inhomogeneous composites in which both compositional profile and properties vary smoothly in one or more preferred direction(s). Many favorable performances in engineering applications offered by FGMs such as high resistance to large temperature gradients and reduction of stress concentration have prompted more investigations on their behavior. On the basis of the conventional continuum theory, static and dynamic characteristics of FGM structures subjected to thermomechanical loads have been studied in the past two decades. FGMs have recently attracted even more attentions for their application in micro-electro-mechanical systems [3–6], where microscale structures play a significant role. In such scales, however, it is experimentally understood that a classical treatment is no longer adequate due to a phenomenon often called size effect. That is, discontinuities of materials manifest themselves in structural behaviors. On the other hand, traditional laminated PMs suffer from several disadvantages such as stress concentration near the inter-layer surfaces [7] or creeping at high temperature [8]. In order to remedy the deficiencies of laminated PMs, functionally graded piezoelectric materials (FGPMs) are developed.

Although size effect observed experimentally in the past two decades [9–14], theoretical extension of conventional continuum theory can be traced back to 1960s when couple stress theory (CST) was proposed [15–17]. The theory received little attention due to the inclusion of asymmetric couple stress tensor, giving rise to the involvement of two non-classical constants for isotropic materials [18,19]. The modified version of couple stress theory (MCST), proposed by Yang et al. [20], removes this difficulty through introducing an additional equilibrium equation. Consequently, only one new constant, the so called material length scale parameter, appears in the theory. Ma et al. [21–23], Tsiatas [24], Asghari et al. [25] and Akgoz and Civalek [26] were pioneers to present size-dependent models for bending, vibration and buckling of beams and plates. Ashoori and Mahmoodi [27] developed microstructure-dependent model of rectangular plates for bending analysis. Employing a meshless method, Roque et al. [28] studied microscale Mindlin plates. In an interesting work due to Ghayesh et al. [29], the nonlinear dynamics of beams is investigated according to MCST in which Galerkin method along with cylindrical arc-length technique are utilized. In the category of circular plates, axisymmetric nonlinear bending of FGM plates is

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formulated by Reddy and Berry [30]. Moreover, a comprehensive study on bending, free vibration and buckling of annular size-dependent Mindlin plates composed of FGMs is presented by Ke et al. [31] and Ansari et al. [32]. Very recently, Ashoori and Sadough [33] studied the asymmetric buckling of annular microscale FG plates resting on an elastic medium analytically. In the cases that curvilinear coordinates are unavoidable, the procedure of derivation of governing equations as well as associated boundary conditions is an extremely tedious task. In order to fill this gap, Ashoori and Mahmoodi [34] derived MCST in general curvilinear coordinates. It is noteworthy that such formulation is of interest due to a variety of applications such as problems of cylindrical and spherical cavity expansion in solids and the analysis of asymptotic crack tip field.

In order to complete the mentioned sequential works on the subject, the present work aims to provide an investigation of thermo-electrical stability of circular size-dependent FGPM plates. On the basis of virtual displacement principle, nonlinear governing equations and boundary conditions of circular size-dependent FGPM plates under thermoelectrical loading are extracted. These equations are then exploited to study the nonlinear thermoelectrical bending and post-buckling and also snap-through phenomenon due to lateral loading of thermally preloaded plates. Geometrical nonlinearity is taken into account in the sense of von-Karman nonlinearity. Each thermomechanical property of FGPM plates is assumed to vary in the thickness direction based on a power law form. Furthermore, two types of thermal loading including uniform temperature rise and heat conduction through the thickness as well as two cases of boundary conditions, including clamped and simply supported, are considered. Ritz finite element method is employed to obtain discrete form of equilibrium equations. The matrix representation of governing equations are solved with the help of three different techniques. The response of circular size-dependent FGP plates is either bifurcation type buckling or nonlinear bending. In the cases that bifurcation type instability may occur, the direct displacement control method is employed. In other cases, the response is of nonlinear bending type to which standard load control strategies are appropriate. Newton-Raphson method is therefore used for this purpose. It is clear that thermally deformed circular size-dependent FGP plates with immovable in-plane boundary condition are susceptible to snap-through instability due to lateral loading opposite to the direction of deflection. On the other hand, it is well known that standard load control methods fail to trace nonlinear equilibrium paths through limit points and thus path following methods must be employed instead. Among the existing path following solution methods, the cylindrical (pseudo) arc-length method is utilized in this work. The numerical results of this study are first justified by available data on open literature. Various parametric studies are then presented to investigate the influence of involved parameters.

2. Governing equations

Consider a solid circular plate composed of FGPMs of thickness h, radius a, and referred to the cylindrical coordinates (r, θ, z). The plate is subjected to an external voltage Φ_0, the field of temperature rise Θ, and lateral load q. According to a power law for volume fraction of constituents together with Voigt's rule, a typical property of the FGPM plate, such as P, is written as

$$P(z) = P_l + P_{ul}\left(\frac{1}{2} + \frac{z}{h}\right)^k, \quad P_{ul} = P_u - P_l \tag{1}$$

in which P_l and P_u represent the corresponding properties of the lower and upper surfaces, respectively, and k is the non-negative power law index. All thermo-electro-mechanical properties encountered here are assumed to be graded in the thickness direction according to (1).

Loading conditions and response of the plate are considered to be axisymmetric in the present work. Moreover, the classical plate theory, appropriate to the thin class of plates, is employed. Thus the displacement field of the FGP plate is

$$u_r(r, z) = u(r) - z\frac{\mathrm{d}w}{\mathrm{d}r}u_z(r, z) = w(r) \tag{2}$$

Similar to the displacement field, electric potential is required to be lumped in the thickness direction. Thus the electric potential distribution is assumed to be [35,36]

$$\Phi(r, z) = \cos\left(\frac{\pi z}{h}\right)\phi(r) + \frac{z}{h}\Phi_0 \tag{3}$$

where $\phi(r)$ denotes the electric potential of mid-plane to be determined. Care must be taken that the above form satisfies closed-circuit electric conditions on the upper and lower surfaces.

Taking into account von-Karman assumptions, the nonzero components of strain and curvature tensors are

$$\varepsilon_r = \frac{\mathrm{d}u}{\mathrm{d}r} - z\frac{\mathrm{d}^2w}{\mathrm{d}r^2} + \frac{1}{2}\left(\frac{\mathrm{d}w}{\mathrm{d}r}\right)^2 + \frac{\mathrm{d}w}{\mathrm{d}r}\frac{\mathrm{d}w^*}{\mathrm{d}r}\varepsilon_\theta = \frac{u}{r} - \frac{z}{r}\frac{\mathrm{d}w}{\mathrm{d}r}\chi_{r\theta} = \frac{1}{2}\left(-\frac{\mathrm{d}^2w}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}w}{\mathrm{d}r}\right) \tag{4}$$

in which w^* denotes initial geometrical imperfection. On the fact that the electric field equals to the negative gradient of the electric potential, (3) is rewritten as

$$E_r = -\cos\left(\frac{\pi z}{h}\right)\frac{\mathrm{d}\phi}{\mathrm{d}r}E_z = \frac{\pi}{h}\sin\left(\frac{\pi z}{h}\right)\phi(r) - \frac{\Phi_0}{h} \tag{5}$$

Within the linear framework and also after the appropriate replacement of Lame constants by the coefficients of plane stress state, the constitutive relations for a material with z-axis polarization are given by

$$\begin{Bmatrix}\sigma_r\\ \sigma_\theta\\ \mu_{r\theta}\end{Bmatrix} = \begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & 2Q_{66}\ell^2\end{bmatrix}\left(\begin{Bmatrix}\varepsilon_r\\ \varepsilon_\theta\\ \chi_{r\theta}\end{Bmatrix} - \Theta\begin{Bmatrix}\alpha\\ \alpha\\ 0\end{Bmatrix}\right) - \begin{bmatrix}0 & e_{31}\\ 0 & e_{31}\\ 0 & 0\end{bmatrix}\begin{Bmatrix}E_r\\ E_z\end{Bmatrix}\begin{Bmatrix}\mathcal{D}_r\\ \mathcal{D}_z\end{Bmatrix} = \begin{bmatrix}0 & 0 & 0\\ e_{31} & e_{31} & 0\end{bmatrix}\begin{Bmatrix}\varepsilon_r\\ \varepsilon_\theta\\ \chi_{r\theta}\end{Bmatrix} + \begin{bmatrix}\epsilon_{11} & 0\\ 0 & \epsilon_{33}\end{bmatrix}\begin{Bmatrix}E_r\\ E_z\end{Bmatrix} + \Theta\begin{Bmatrix}0\\ p\end{Bmatrix} \tag{6}$$

where $\boldsymbol{\mu}$ is symmetric couple stress tensor and ℓ represents the so called material length scale parameter. Furthermore, α, e_{31}, ϵ_{ij} and p denote the coefficient of thermal expansion, piezoelectric modulus, dielectric coefficients and pyroelectric constant, respectively.

In order to extract governing equations, the static version of virtual displacement principle is utilized which for size-dependent FGPM plates occupying region of volume $\mathcal{V}$ subjected to thermoelectrical loading states that the variation of electric total Gibbs energy equals to the virtual work done on the plates, i.e.,

$$\delta\mathcal{G} = \delta\mathcal{W} \tag{7}$$

in which

$$\delta\mathcal{G} = \int_{\mathcal{V}}(\sigma_{ij}\delta\varepsilon_{ij} + \mu_{ij}\delta\chi_{ij} - \mathcal{D}_i\delta E_i)\mathrm{d}\mathcal{V} \tag{8}$$

Substitution of (4) and (8) into (7) yields

$$\int_0^a q\delta w\ r\mathrm{d}r = \int_0^a r\mathrm{d}r\int_{-\frac{h}{2}}^{\frac{h}{2}}(\sigma_r\delta\varepsilon_r + \sigma_\theta\delta\varepsilon_\theta + 2\mu_{r\theta}\delta\chi_{r\theta} - \mathcal{D}_r\delta E_r - \mathcal{D}_z\delta E_z)\mathrm{d}z = \int_0^a\left[N_r\left(\frac{\mathrm{d}\delta u}{\mathrm{d}r} + \frac{\mathrm{d}w}{\mathrm{d}r}\frac{\mathrm{d}\delta w}{\mathrm{d}r} + \frac{\mathrm{d}w^*}{\mathrm{d}r}\frac{\mathrm{d}\delta w}{\mathrm{d}r}\right) - M_r\frac{\mathrm{d}^2\delta w}{\mathrm{d}r^2} + \frac{N_\theta}{r}\delta u - \frac{M_\theta}{r}\frac{\mathrm{d}\delta w}{\mathrm{d}r} + \mathfrak{M}_{r\theta}\left(-\frac{\mathrm{d}^2\delta w}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}\delta w}{\mathrm{d}r}\right) + \overline{\mathcal{D}}_r\frac{\mathrm{d}\delta\phi}{\mathrm{d}r} - \overline{\mathcal{D}}_z\delta\phi\right]r\mathrm{d}r \tag{9}$$