Thin–Walled Structures 111 (2017) 29–37

Contents lists available at ScienceDirect

Thin–Walled Structures

journal homepage: www.elsevier.com/locate/tws

Full length article

Stress analysis for the orthotropic pressurized structure of the cylindrical shell and spherical head

Chunxiao Li, Tianyu Yao, Xiaohua He*, Changyu Zhou*

School of Mechanical and Power Engineering, Nanjing Tech University, Nanjing 211816, China

ARTICLE INFO

Keywords:
Orthotropy
Cylindrical shell
Spherical head
Stress

ABSTRACT

According to anisotropic shell theory, the pressurized structure of cylindrical shell and spherical head is analyzed, and the shear force, bending moment and stress solution at the juncture are obtained. The influence of anisotropic mechanical properties and geometric parameters on shear force, bending moment and stress distribution along the cylindrical shell is discussed, and the results are compared with the isotropic results. Additionally, Hill48 anisotropic yield criterion is also used to derive the equivalent stress intensity expression of the plane stress state, and the effect of thickness ratio and elastic modulus ratio on Hill48 equivalent stress at the juncture is discussed. Finally, finite element method is applied to verify the results of stress intensity calculation, which shows that the finite element solution and the analytical solution are in good agreement.

1. Introduction

In the pressure vessels, film stress distributes evenly along the wall of the spherical head, equally, and uniformly. At the same time, spherical head is easy to manufacture, widely applied as the middle or low pressure vessel. But the wall thickness, load, temperature and mechanical properties along the axial direction of shell may be abrupt for the combined structure, and these factors can be expressed as the discontinuity of pressure vessel structure [1]. It is worth noting that the juncture between cylindrical shell and spherical head is not continuous, which may cause a large local stress. For accurately understand the stress distribution of spherical head, Fu [2] according to the non-moment theory deduced the stress expression of spherical head, and the stress distribution law was studied by experiment method and ANSYS finite element analysis. Awrejcewicz [3] estimated and predicted a critical set of critical parameters responsible for buckling of spherical circle axially symmetric shells. The buckling phenomenon under static loading was investigated. Cui [4] solved the basic governing differential equations for conical shells by performing magnitude order analysis and neglecting the quantities with h/R magnitude order with a simple and accurate solution for conical shells derived by solving the second-order differential equation. Rilo [5] analyzed the solutions of displacements and stresses in spherical heads over rectangular areas based on spherical shallow shell equations, and represented the displacement and loading terms by use of double Fourier series expressions. Awrejcewicz [6] analyzed stress-strain state of the laminated shallow shells under static loading by using the R-functions theory together with the spline-approximation, and the comparison of obtained results with those results by using ANSYS were also presented.

For the problem of vibration of cylindrical shell, A.V. Krysko [7] solved strongly nonlinear partial differential equations by modeling the dynamics of closed circular shells using the Bubnov–Galerkin approach, V.A. Krysko [8] studied the complex vibrations of closed cylindrical shells of infinite length and circular cross-section subjected to transversal local load.

At present, the stress analysis and strength evaluation of pressurized structure are usually carried out by using the theory of isotropic mechanics. However, material in the rolling process will appear a texture phenomenon and anisotropy such as titanium which is widely used in recent years. Therefore, it is necessary to carry out detailed stress analysis on the anisotropic pressurized structure to meet the requirements of strength and stiffness in different directions.

For the anisotropic structures, Li [9] analyzed the influence of anisotropy on the internal forces and moments of the laminated conical shells. According to the principle of energy deformation and the constitutive relation of orthotropic materials, Zeng [10] derived the elastic-plastic stress expression of the axially loaded cylindrical shell under the condition of simply supported at both ends. Rotter [11] combined the generalized Hooke's law, the axial film stress resultant force and the axial bending moment, and developed the complete solution equation of the bending theory for cylindrical shell under different pressure and axial loads. Chandrashekhara [12] applied the theory of elasticity, the classical shell theory and the shear deformation

* Corresponding authors.
E-mail addresses: xh_he@njtech.edu.cn (X. He), changyu_zhou@163.com (C. Zhou).

http://dx.doi.org/10.1016/j.tws.2016.11.003
Received 5 March 2016; Received in revised form 3 November 2016; Accepted 8 November 2016
0263-8231/

theory to study the problem of infinite transversely isotropic cylindrical shell under the action of axial symmetrical radial load. The influence of anisotropy and the diameter thickness ratio of shell on the stress and displacement distribution was shown through some typical results. Kirichenko [13] established the mathematical modeling of evolutionary states of non-homogeneous multi-layer shallow shells with orthotropic initial imperfections.

There are still many scholars to study the stress analysis at the juncture between cylindrical shell and head. The discontinuous stresses of the anisotropic cylindrical shell and the connection of the different head were analyzed by Paliwal [14], the corresponding discontinuous stress and displacement expressions are obtained. Mechanical analysis of the connection structure of the pressure cylindrical shell and the flat plate was made by Gao [15], the stress expression at the dangerous point of the structure were obtained, and the stress distribution of the joint was similar to that of the isotropic one, but the peak stress difference was higher which was associated with the E_x/E_θ. Through the analysis of the stress on the small end of orthotropic conical shell, Yao [16] obtained shear force, bending moment and stress solution at the small end of conical shell.

In present paper, by analyzing the force and bending moment at juncture of the orthotropic cylindrical shell and the spherical head, the stress expression is obtained, and the impact of elastic modulus ratio and thickness ratio on the stress distribution is discussed. According to Hill48 anisotropic yield criterion, the equivalent stress intensity expression of the plane biaxial stress state is obtained, and the effect of thickness ratio and elastic modulus ratio on Hill48 equivalent stress at the juncture is discussed at the same time. The calculation results of the stress intensity are verified by finite element results which provide a reference for the design of the orthogonal anisotropic pressurized structure.

2. Mechanics analysis of the connection between orthotropic cylindrical shell and spherical head

Under the influence of internal pressure, the stress of spherical head (2) and cylindrical shell (1) is shown in Fig. 1. The inner pressure is p, the radius of cylindrical shell and spherical head is R, the thickness of cylindrical shell and spherical head are respectively t_1 and t_2. To solve edge load shear force and bending moment, the deformation caused by the load at the edge of its positive and negative direction are shown in Fig. 1: Radial displacement pointing to the rotation shaft is as the positive direction. The left layout of Fig. 1 is used for determining rotation direction and the counter clockwise is positive.

On the edge of the cylindrical shell, shear force Q_0 and bending moment M_0 are applied along the circumference direction. The basic differential equation of the orthotropic cylindrical shell by moment theory [17] is:

$$\frac{d^4w}{dx^4} + 4\beta_1^4 w = 0 \tag{1}$$

where $\beta_1 = \sqrt[4]{\frac{3E_\theta(1-\nu_\theta\nu_x)}{E_x R^2 t_1^2}}$.

The general solution for the homogeneous equation:

$$w = e^{\beta_1 x}(C_1\cos\beta_1 x + C_2\sin\beta_1 x) + e^{-\beta_1 x}(C_3\cos\beta_1 x + C_4\sin\beta_1 x) \tag{2}$$

In the formula, C_1, C_2, C_3 and C_4 are integral constants, which are determined by the boundary conditions of cylindrical shell.

When the cylindrical shell is long enough, with the increase of x, the bending deformation gradually decay and disappear, so the $e^{\beta_1 x}$ in formula (2) is close to 0, that is, C_1=C_2=0, so the formula (2) can be written as:

$$w = e^{-\beta_1 x}(C_3\cos\beta_1 x + C_4\sin\beta_1 x) \tag{3}$$

At the juncture of cylindrical shell and spherical head, that is $x = 0$, boundary conditions are as below:

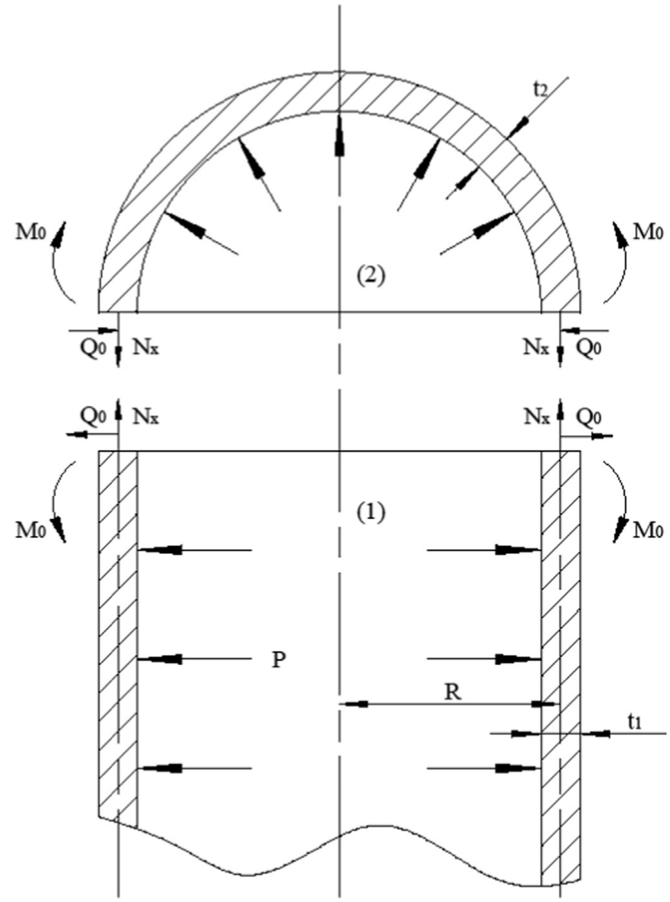

Fig. 1. Load at the junction of cylindrical shell and spherical head.

$$M_x|_{x=0} = -D_1\left(\frac{d^2w}{dx^2}\right)_{x=0} = M_0 \tag{4}$$

$$Q_x|_{x=0} = -D_1\left(\frac{d^3w}{dx^3}\right)_{x=0} = Q_0 \tag{5}$$

Substituting the boundary conditions into the differential equation:

$$w = \frac{e^{-\beta_1 x}}{2\beta_1^3 D_1}[\beta_1 M_0(\sin\beta_1 x - \cos\beta_1 x) - Q_0\cos\beta_1 x] \tag{6}$$

where $D_1 = \frac{E_x t_1^3}{12(1-\nu_\theta\nu_x)}$ is the flexural modulus.

The maximum displacement and rotation angle at $x = 0$ are :

$$(w)_{x=0} = -\frac{1}{2\beta_1^2 D_1}M_0 - \frac{1}{2\beta_1^3 D_1}Q_0 \tag{7}$$

$$(\phi)_{x=0} = \left(\frac{dw}{dx}\right)_{x=0} = \frac{1}{\beta_1 D_1}M_0 + \frac{1}{2\beta_1^2 D_1}Q_0 \tag{8}$$

For cylindrical shell, the displacement and rotation angle caused by shear force and bending moment are as below.

$$\Delta_1^{M_0} = -\frac{1}{2\beta_1^2 D_1}M_0 \tag{9}$$

$$\Delta_1^{Q_0} = -\frac{1}{2\beta_1^3 D_1}Q_0 \tag{10}$$

$$\varphi_1^{M_0} = \frac{1}{\beta_1 D_1}M_0 \tag{11}$$

$$\varphi_1^{Q_0} = \frac{1}{2\beta_1^2 D_1}Q_0 \tag{12}$$

The basic differential equation of orthotropic spherical head by