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Buckling of spherical shells subjected to external pressure: A comparison of experimental and theoretical data

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ABSTRACT

This paper focuses on spherical shells under uniform external pressure. Ten laboratory scale models, each with a nominal diameter of 150 mm, were tested. Half of them were manufactured from a 0.4-mm stainless steel sheet, whereas the remaining five shells were manufactured from a 0.7-mm sheet. The geometry, wall thickness, buckling load, and final collapsed mode of each spherical shell were measured, as well as the material properties of the corresponding sheet. The buckling behaviors of these shells were demonstrated analytically and numerically according to experimental data. Analyses involved considering the average geometry, average wall thicknesses, and average elastic material properties. Numerical calculations entailed considering the true geometry, average wall thicknesses, and elastic-plastic modeling of true stress–strain curves. Moreover, the effects of purely elastic and elastic-perfectly plastic models on the buckling loads of spherical shells were examined numerically. The results of the experimental, analytical, and numerical investigations were compared in tables and figures.

1. Introduction

For more than 100 years, research has been published on spherical shells subjected to uniform external pressure. Knowledge about this type of loading has been widely applied in various engineering fields such as those involving underwater pressure hulls, underground pressure vessels, and underpressure tanks [1,2]. In particular, the spherical configuration is broadly considered an ideal structure for the pressure hulls of deep submersibles. This is due to the extremely efficient stress and strain distributions in the material [3,4]. Although the theoretical elastic buckling loads are high, the spherical shells have been found to be highly imperfection sensitive and be strongly affected by plastic material properties. The experimental buckling loads are even lower than the theoretical ones.

The difference between theory and experiment has prompted numerous studies regarding the buckling of spherical shells loaded by external pressure. For example, Blachut et al. presented a series of experimental and numerical studies concerning elastic-plastic buckling of spherical [5], torispherical [6], conical [7,8] or barreled shells [9,10] under external pressure. In most of their studies, the material was assumed to be elastic-perfectly plastic. The effects of initial geometric imperfections were also taken into account, which included eigenmode imperfections derived from linear elastic buckling analysis of the perfect geometries or deterministic ones derived from a limited number of measuring points. Moreover, Pan et al. performed a set of experimental and numerical studies on the ultimate strength levels of spherical pressure hulls used in deep submersibles [11,12]. Their numerical models were elaborated on the basis of initial equivalent geometric imperfections in the shape of the first eigenmode and a local dimple. Their numerical predictions were verified by pressurizing four laboratory scale spherical hulls to collapse. More recently, Quilliet carried out elasticity theory calculations to predict the collapsed mode of a spherical shell [13]. Quillet's prediction resembled previously published experimental results. However, little attention has been paid to true geometry, including deterministic imperfections, derived from a more precise measurement. And the effects of plastic material properties including or excluding yield strengths and hardening parameters on the buckling of spherical shells were rarely investigated as well. Further study is still necessary in this branch of mechanics.

For investigating the buckling of spherical shells loaded with external pressure, spherical shells were manufactured from 304 stainless steel sheets through stamping and butt-welding processes. The geometric and buckling properties of these spherical shells were demonstrated by a series of tests. The buckling and postbuckling behaviors were determined numerically and verified experimentally. The numerical analysis was based on deterministic imperfections

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obtained from measured geometric shapes and elastic-plastic modeling of true stress–strain curves. Furthermore, the effects of constitutive models, such as purely elastic and elastic-perfectly plastic modeling of material, on the buckling loads were studied numerically. This paper aims to provide a rational approach to predicting the real load-carrying capacities of spherical shells.

2. Materials and methods

This study involved sampling and analyzing 10 spherical shells to determine their buckling behaviors. A series of tests were performed to obtain the geometric and buckling properties of these shells in addition to their material properties.

2.1. Shell manufacturing and testing

Each spherical shell was manufactured using the tungsten inert gas butt welding of two coupled hemispherical shells, after which the excess of the weld has been removed by grinding and then been polished. Each hemispherical shell was cut and stamped from 304 thin stainless steel sheets with a nominal thickness of either 0.4 mm or 0.7 mm. Ten spherical shells with a nominal diameter of 150 mm were manufactured for the tests. Five of them were fabricated from a 0.4-mm-thick sheet and were denoted as t0.4-1, t0.4-2, t0.4-3, t0.4-4, and t0.4-5. Five other shells were fabricated from a 0.7-mm-thick sheet and were denoted as t0.7-1, t0.7-2, t0.7-3, t0.7-4, and t0.7-5. In addition, all the shells were not stress relieved during the manufacturing process because the ratios of the wall thickness to the nominal diameter were very low. Before the spherical shells were tested, the wall thickness and geometric shape were measured for all the shells.

First, the thickness of each wall was measured using an ultrasonic probe at 13 equidistant points along a meridian for eight equally spaced meridians, as detailed in Fig. 1. Each shell was measured at $8\times 11+2=90$ points. The values of the minimum (t_{min}), maximum (t_{max}), and average wall thicknesses (t_{ave}), as well as the corresponding standard deviations (t_{std}), are listed in Table 1. Overall average wall thickness profile from the North-Pole to the South-Pole for a t0.4-1 spherical shell and its thicknesses of all measure points are also showed in Fig. 2 and Table 2 respectively. The average variation between the maximal and minimal wall thicknesses was approximately 17%, which may be attributed to the stamping process. Second, the geometries of all the spherical shells were obtained using a three-dimensional optical scanner, developed by Open Technologies Corporation. The scanned accuracy is not more than 0.02 mm reffering to operating manual provided by the corporation. Each shell surface was scanned in the form of a point cloud and automatically transformed into a CAD model. Each model demonstrated the real geometric shape of the corresponding shell, which contained deterministic geometric imperfections caused by manufacturing processes. Furthermore, the minimum (r_{min}), maximum (r_{max}), and average radii (r_{ave}) of each shell were also obtained from the CAD model in addition to the corresponding standard deviations (r_{std}); these values are listed in the final four columns of Table 1, while contours of local radii of curvature for a t0.4-1 spherical shell are presented in Fig. 3.

The spheres were empty and they were floating in the test vessel. This floating was expected to exert a strong influence on the buckling behavior of spherical shells. The net buoyancy values of the spherical shells were considerably high because their buoyant loads were higher than their dead-weight values. The net buoyancy was obtained by:

$$F_{\mathrm{netbuoyant}} = \frac{4}{3}\pi \mathrm{r}^3 g\rho_{water} - \frac{4}{3}\pi\left(r^3 - (r-t)^3\right) g\rho_{steel}, \tag{1}$$

where, r, t are the nominal radius and nominal thickness of a spherical shell, ρ_{steel} is the density of strainless steel, ρ_{water} is the density of water inside the vessel, g is the gravitational acceleration. Assume that: $r=75$ mm, $t=0.4$ mm for a t=0.4-mm spherical shell, $t=0.7$ mm for a t=0.7-mm spherical shell, $\rho_{steel}=7930\ \mathrm{kg/m^3}$, $\rho_{water}=1000\ \mathrm{kg/m^3}$, $g=9.8\ \mathrm{m/s^2}$. Therefore, the net buoyancy of a $t=0.4$-mm spherical shell is 15.12N, whereas the results of calculation can approximately be 13.50N for a $t=0.7$-mm spherical shell. To minimize this effect, each spherical shell was encased in a string bag connected to a ballast pig. The weight of the pig was slightly higher than the buoyant load of the spherical shell. The shell, bag, and pig were then immersed together in a cylindrical pressure chamber with a 200-mm inner diameter, 400-mm total length, and 20-MPa maximum pressure. The chamber (located at Jiangsu University of Science and Technology) entailed using water as a pressurizing medium. The pressure inside the chamber was controlled automatically by a programmable logic controller and measured using a pressure transducer. All the spherical shells failed suddenly with substantial decreases in pressure. Thus, determining the buckling load was very simple.

2.2. Material properties

In cases of uniform external pressure, the buckling behaviors of spherical shells are determined according to the compression stress–strain behavior of the relevant material. However, experiments to demonstrate such behaviors with thin-walled structures are extremely difficult to conduct. Therefore, the compression behavior of steel is assumed to be the same as its tension behavior. This hypothesis has been frequently used in the buckling prediction of various shells of

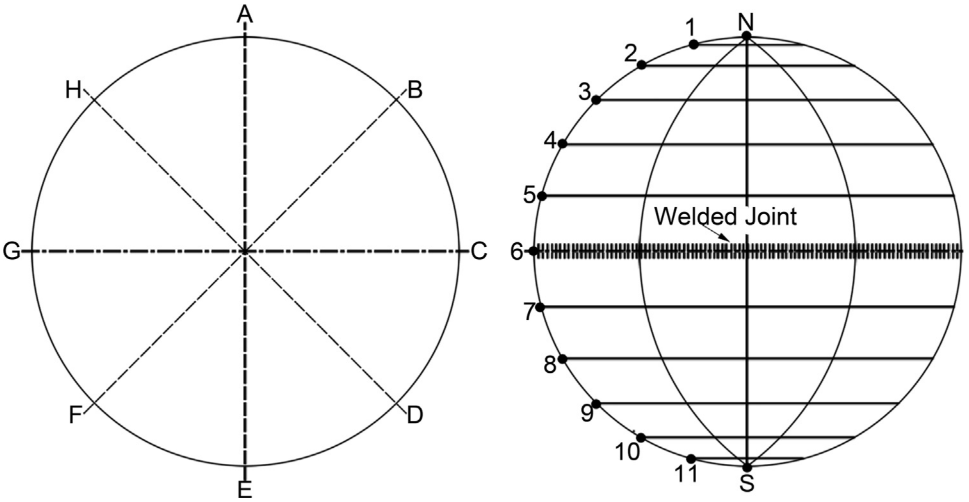

Fig. 1. Typical distribution of testing points for wall thickness.