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Natural frequencies and buckling of compressed non-symmetric thin-walled beams

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ABSTRACT

The experimental natural frequencies and buckling loads of aluminium slightly non-symmetric thin-Walled beams under axial force are compared with numerical results. A universal machine compressed the specimens by providing their ends with a relative axial displacement; suitable end constraints for warping were manufactured; PZT pickups, which revealed to be reliable in this field, helped extracting the natural frequencies under various compressive forces. An *in-house* code provided the corresponding numerical results, accounting for the effects of non-symmetric cross-sections and warping. The comparison provides a good agreement between the two sets of results, showing that the predictions of the code are verified by experiments, thus opening the way to possible applications in inverse problems.

1. Introduction

Thin-walled beams with open cross-sections exhibit bending mixed with torsion, which turn uncoupled only when symmetries are present [1], and their negligible torsion stiffness makes their post-buckling behaviour unstable in general [1]. Moreover, boundary effects do not quickly vanish along the beam length according to Saint-Venant's principle [2]: thus, warping and warping constraints may become meaningful, providing an addend to Saint-Venant's torsion couple [2,3]. Since thin-walled open profiles are of common use in engineering, the investigation of their static and dynamic stability is of interest. The authors have published papers on the matter from various points of view [4–11], which will be shortly outlined. As for the static stability of compressed thin-walled open profiles and the possible occurrence of buckling, one usually assumes that the load does not deform the beam (trivial fundamental path) [1–3]. However, such an assumption is meaningful for bulky profiles with high axial stiffness, while in general a non-trivial pre-buckling path cannot be neglected for thin-walled open profiles due to the coupling of bending and torsion, and to the different orders of magnitude of the relevant rigidities. Thus, investigations on how non-trivial pre-buckling paths may influence the critical compressive load are interesting. Numerical studies [7–9] were performed by means of an *in-house* finite differences code based on the model described in [4–6] and refined in [10]. Numerical and experimental investigations on the effect of compressive loads on the natural frequencies of compressed beams are in [11].

In the international literature, the studies on the effect of non-trivial pre-buckling paths date back at least to the work of the research group of Trahair [12–15]. They investigated, by means of a finite element technique based on a suitable formulation of the elastic energy, the effects of pre-buckling deformations for mono-symmetric thin-walled open beam-columns, including the effects of second-order moments due to the pre-buckling displacements and the axial loads. Erkmen and Attard [16] recently took up this work and studied the lateral-torsion buckling of thin-walled open profiles, by a geometrically nonlinear formulation accounting for shear deformations, and a finite element technique with an incremental-iterative procedure to follow the pre-buckling and the post-buckling paths. The research group of Mohri and Potier-Ferry also tackled the problem [17,18] by adopting numerical techniques based on a beam model following the well-known works by Vlasov [3] and Wagner [19], accounting for the coupling between bending and torsion, considering I and T sections. Andrade and Camotim performed analogous research [20] by means of the Rayleigh–Ritz method, using trigonometric functions to approximate the beam critical buckling mode, and investigating mono- and two-symmetric sections.

The effects of an axial load on the natural vibration of beams is known since the paper by Woynowski-Krieger [21], investigated also by Bishop and Price [22], Bokaian [23,24] and Stephen [25]: roughly speaking, an axial load perturbs the material stiffness of the beam by adding a geometric contribution, influencing the natural frequencies of the element. Tensile forces tend to increase the total stiffness of the

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element, compressive forces act in the opposite sense: they tend to diminish the natural frequencies of a beam until they reach zero, inducing static buckling, see for instance Bazant and Cedolin [26].

In order to investigate the static buckling of compressed beams in a general dynamic setting according to Lyapounov we may thus arrange, from the points of view of numerical simulations and experiments, a load path where the applied compressive load is gradually increased. Then, suitable acquisition devices detect the first *n* natural frequencies. As the load grows, the global stiffness of the beam goes down, due to the geometric effect of the axial force, and the first *n* natural frequencies diminish, until the *i*-th ($i = 1, 2, \ldots, n$) critical load is attained when the *i*-th natural frequency vanishes.

Since we already investigated some thin-walled open profiles, both numerically and experimentally, the next step was to design an experimental campaign of validation of previous results, with the aim of: a) generalizing previous experimental investigations, now accounting for warping and warping constraints, that are known to characterize thin-walled profiles; this was done by designing and realizing suitable end constraint devices to match the analytical conditions introduced in the numerical investigation; b) finding experimental validation of the numerical results obtained so far; and c) highlighting, by experimental evidence, the role of the pre-buckling equilibrium path, usually neglected in the literature.

It is apparent that this is a long-term project, since it involves: a) numerical simulations, plus possible refinements both of the adopted beam model and of the numerical technique; b) design and realization of experimental settings and suitable data acquisition systems, devoted to the particular case; c) analysis and numerical treatment of the experimental data in order to obtain reliable results; d) possible issues in all the preceding fields that have to be solved to proceed further in the investigation.

The research group began investigating the effect of compressive loads operated by a universal MTS testing machine on simple specimens, suitably constrained at the ends by devices designed and made on purpose, to take into account the warping properties of thin-walled profiles. Some of the authors adopted laser techniques [11] to detect the natural frequencies of the tested beams, which were symmetric and had been suitably curved to simulate a known imperfection, or a pre-buckling path. In subsequent papers [27,28] we showed how the use of standard PZT pickups of musical technologies is effective, precise, robust, and reliable in the experimental campaign to detect the natural frequencies of the considered specimens with symmetric cross-sections.

In this contribution, we extend our investigation, still on the basis of the model of thin-walled open profiles presented, investigated and discussed in some of our previous works [4–11,27,28]. Indeed, once again we present experimental and numerical results about the natural frequencies and the corresponding buckling loads for an open thin-walled profile suitably compressed by a universal testing machine. This specimen, however, is different from those investigated by the authors so far, in that it exhibits a slightly non-perfectly symmetric cruciform cross-section. The results for the corresponding symmetric cruciform cross-section, which is known to have null warping stiffness, are in the literature since the pioneering works by Timoshenko [2], and will be a benchmark for comparison, also from the point of view of the numerical results provided by our *in-house* code. In this contribution, at first we describe how the effects of geometrical non-symmetries are detected experimentally. Then, the obtained results in terms of both natural frequencies and buckling loads are thoroughly presented and discussed, and comparisons with the corresponding numerical ones are provided. We try to highlight the key role of geometrical non-symmetries to the detected physical quantities, and to provide them with a reasonable physical meaning.

Other specimens with doubly symmetric cross-section and non-negligible warping stiffness are now considered in the experimental and numerical campaign, and will be dealt with in a future contribution.

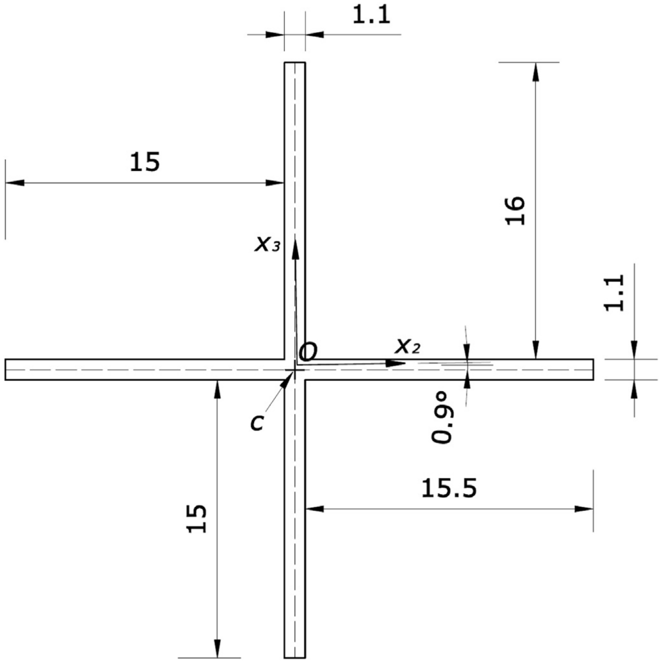

Fig. 1. Cross-section of the specimen (dimensions in mm).

2. Experimental setup, equipment, and testing procedure

As a first step, a suitable experimental setup was designed, calibrated, and validated. To this aim, a set of specimens endowed with cruciform cross-section was chosen and bought at a specialized manufacturer. Their cross-section is shown in Fig. 1, and their physical and mechanical properties are listed in Table 1.

The geometrical quantities and mass density were correctly measured, within the limits of the instruments at ease and the precision of the personnel involved in the tests. On the other hand, supposing the specimen to be composed of a linear elastic homogeneous and isotropic material, its Young's modulus and Poisson's ratio were assumed on the basis of the features declared by the manufacturer.

The surface of the specimens was not subjected to rectification technological processes: thus, some small variations in the geometrical dimensions of the cross section were present along the beam axis.

Fig. 2 shows the experimental setup, similar to that of other tests performed by the authors, but with new and important features both in the end constraints and in the sensors used to measure natural frequencies.

A servo-hydraulic universal MTS testing machine, with a closed-loop electronic control and a maximum loading capacity of 100 kN, compressed the specimen, which was placed with the axis along the vertical and constrained at its ends to the testing machine through new suitable connections. The latter, shown in Fig. 2a, were designed and realized on purpose in order to control displacements, rotations and twist of the end sections. In detail, they were arranged by manufacturing two brass elements so to allow the insertion of the beam ends, with the possibility of either preventing or permitting the warping of the end

Table 1
Physical and mechanical properties of the specimen.

Unconstrained length, mm	Total web widths, mm	Web thickness, mm	Young's modulus, N m^{-2}	Poisson's ratio, -	Mass density, kg m^{-3}
670	32.1, 31.6	1.1	$70\ 10^9$	0.3	2600