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Nonlinear static analysis of an underwater elastic semi-toroidal shell

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ABSTRACT

This paper addresses the nonlinear static behavior of an elastic semi-toroidal shell of circular cross section subjected to external hydrostatic loading by using membrane theory. In this study, semi-toroidal shell geometry can be computed by differential geometry. Energy functional of the semi-toroidal shell can be obtained by principle of virtual work, and written in the appropriate form. The static response of the semi-toroidal shell can be computed by nonlinear finite element method. The nonlinear numerical solutions can be obtained by the iterative method. The effects of sea water depth, cross-sectional radius to thickness, and cross-sectional to bend radii ratios on the displacement responses of the semi-toroidal shell are investigated in this paper. Interestingly, it is found that the maximum value of the displacement responses on the semi-toroidal shell under hydrostatic loading occurred to the toroid internal crest.

1. Introduction

Toroidal shells are widely used in many engineering applications, such as vehicle or aircraft tires, rocket fuel tanks, pressure vessel, and circumferential reinforcement for sub-marine hulls [1–4]. In literature, both linear and nonlinear membrane theories are used to solve problems of toroidal shell subjected to internal pressure, examples of these works are described by Flügge [5], Jordan [6], Sanders and Liepins [7], and Peng [8]. The classical shell theory with the differential quadrature method in order to compute the natural frequencies of circular and elliptical cross sectional orthotropic toroidal shells is found in the research works of Wang and Redekop [9], and Xu and Redekop [10]. Since the behavior of toroidal shell is difficult to model using membrane shell theory due to a singularity in the strain-displacement relations occurring at the toroid crest, the new analysis technique coupled with finite element analysis for investigating the behavior of the toroidal shell to insure continuity of displacements is proposed by Vick and Gramoll [11]. Tamadapu and DasGupta [12] studied a finite deformation of hyperelastic circular cross-sectional toroidal membrane under internal pressure by the variational formulation. For the deformation of flat toroidal membrane, this was investigated and reported by Roychowdhury and DasGupta [13]. However, the analysis of toroidal shell using membrane theory can be used to estimate the stresses and deformations when the shell geometry and loading do not change rapidly. Therefore, Zingoni et al. [14] presented the approximate bending solution for solving the shell geometry and loading discontinuity effects at horizontal equatorial plane of elliptic toroidal shell. For compendium of the research work on the liquid-containment shell structures, Zingoni [15] reported the recent works on strength, stability, and dynamics analysis of liquid-containment shell structures with arbitrary shapes including environmental and thermal effects.

Recently, the concept of liquid storage in the underwater offshore field has been of concern and interest in the petroleum operations. Advanced researches for investigating the behavior of underwater shell structures have received much attention from many researchers [16–19]. Jasion and Magnucki [20] proposed the elastic pre-buckling, buckling, and post-buckling states of clothoidal-spherical shaped shell under external pressure by using analytical solution and finite element method. They found that the values of buckling load increase when the geometry of shell goes to a sphere. Based on the related literature in this field, many works focused on the axisymmetric shell with an arbitrary cross-section that intersect the axis of revolution, e.g., spherical and drop shaped shell, except the research work of Wilson et al. [21]. They studied the finite element model of toroidal shell, which is installed in deepwater by assuming fully roller support conditions at intrados and extrados equator on sea bed. However, toroidal shell has a very small thickness when compared with its cross-sectional radius. The bending rigidity has been assumed to be negligible in the present work which allows us to use simple membrane theory [7].

To the authors' knowledge in offshore engineering field, there has not been any study on the nonlinear large displacement analysis of semi-toroidal shell using membrane theory under hydrostatic loading. In this study, the semi-toroidal shell is assumed to be empty or the pressure inside is not considered. The behavior of the semi-toroidal

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shell without internal pressure is similar to the submerged spherical dome in the research work of Wang et al. [18] as well as the other researches on the funicular arches [22–24]. The empty semi-toroidal shell under the hydrostatic loading is sustained in the equilibrium position by the compressive membrane force. The model formulation is based on differential geometry theory [25], including the effects of large displacements and rotations. Most uses of this theory adopted this technique for large displacement analysis [26–29]. Since the semi-toroidal shell geometry is always axisymmetric, any meridional curve are described in polar coordinates. Third-order polynomial shape functions are used in the finite element analysis. Energy functional of the semi-toroidal shell can be obtained by the principle of virtual work [30] and written in the appropriate forms [31] which are introduced in order to reduce the computation time. The numerical results of nonlinear static responses for semi-toroidal shell can be determined by finite element method [32] coupled with the iterative method. It should be noted that the emphasis of the present formulation is to develop a variational model for large deformation analysis of the semi-toroidal shell under external hydrostatic loading, buckling analysis is not yet considered in this study. Although the possibility of buckling of the semi-toroidal shell can occur due to the large value of compressive membrane force. Further investigation on this topic should be carried out in the future.

2. Analytical model formulation

Let R be the bend radius of semi-toroidal shell and a the cross-sectional radius of semi-toroidal shell, as shown in Fig. 1. Then the equation of an axisymmetric circular semi-toroidal shell [25] described in rectangular coordinates (X, Y, Z) is given by

$$(R - \sqrt{X^2 + Y^2})^2 + Z^2 = a^2 \tag{1}$$

where X, Y, and Z are the parametric equation based on the two surface parameters (θ, ϕ) defining the position of a point on the meridian and circumferential lines, respectively, and are given by

$$X = X(\theta, \phi) = (R + a\cos\theta)\cos\phi \tag{2a}$$

$$Y = Y(\theta, \phi) = (R + a\cos\theta)\sin\phi \tag{2b}$$

$$Z = Z(\theta, \phi) = a\sin\theta \tag{2c}$$

These equations are single valued, continuous, and differentiable functions. Furthermore, there shall be a one-to-one correspondence between pairs of two surface parameter (θ, ϕ) values and points on the surface P. For the case of an axisymmetric semi-toroidal shell, any meridional curve may be considered as the generating curve on the $r - Z$ plane, as shown in Fig. 2. Therefore, only the changed shape of

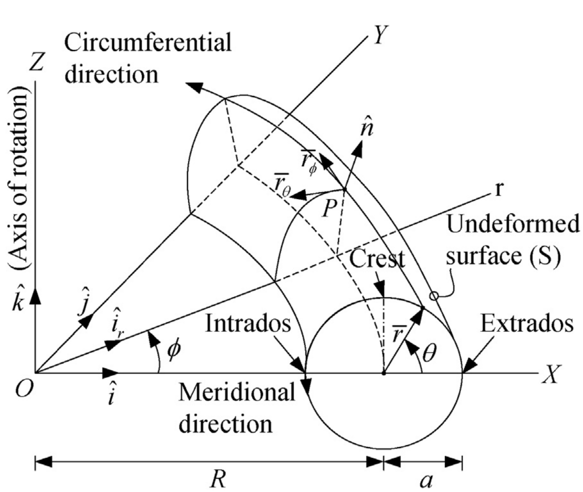

Fig. 1. Geometrical parameters and coordinates of semi-toroidal shell.

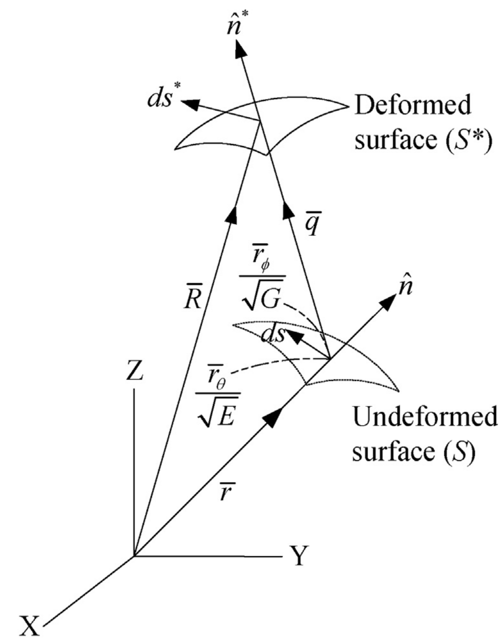

Fig. 2. Position vector field geometry of semi-toroidal shell.

the generating curve needs be considered. Let $\bar{r}$ be the position vector of any point P which can be defined by using parallel circle radius r as

$$\bar{r} = r\cos\phi\,\hat{i} + r\sin\phi\,\hat{j} + Z\,\hat{k} \tag{3}$$

in which

$$r = r(\theta) = R + a\cos\theta \tag{4a}$$

$$Z = Z(\theta) = a\sin\theta \tag{4b}$$

If the semi-toroidal shell is deformed, its undeformed surface (S) moves to a deformed surface (S^*), as shown in Fig. 2. The position vector $\bar{R}$ on the S^* can be determined by the relation

$$\bar{R} = \bar{r} + \bar{q} = \bar{r} + \frac{\bar{r}_\theta}{\sqrt{E}}u + \frac{\bar{r}_\phi}{\sqrt{G}}v + \hat{n}w \tag{5}$$

where u, v, and w are the displacement components in the direction of meridian, circumferential, and normal lines, respectively. Since an axisymmetric semi-toroidal shell problem is considered herein, the circumferential displacement can be neglected herein; that is the term $(\bar{r}_\phi/\sqrt{G})v$ is equal to zero. The unit vector normal to the S at point P is given by

$$\hat{n} = \frac{\bar{r}_\theta \times \bar{r}_\phi}{|\bar{r}_\theta \times \bar{r}_\phi|} = \frac{-rZ_\theta\cos\phi\,\hat{i} - rZ_\theta\sin\phi\,\hat{j} + rr_\theta\,\hat{k}}{D} \tag{6}$$

in which

$$D = \sqrt{EG - F^2} = r\sqrt{r_\theta^2 + Z_\theta^2} \tag{7}$$

Based on differential geometry theory [25], the arc length ds at the S moves to ds^* at the S^* after deformation, which can be obtained by the following equations

$$ds^2 = d\bar{r}\cdot d\bar{r} = Ed\theta^2 + 2Fd\theta d\phi + Gd\phi^2 \tag{8a}$$

$$ds^{*2} = d\bar{R}\cdot d\bar{R} = E^*d\theta^2 + 2F^*d\theta d\phi + G^*d\phi^2 \tag{8b}$$

where $d\bar{r} = \bar{r}_\theta d\theta + \bar{r}_\phi d\phi$ and $d\bar{R} = \bar{R}_\theta d\theta + \bar{R}_\phi d\phi$ are the total differential line elements of the $\bar{r}$ and $\bar{R}$, respectively. It is noted that the subscripts θ and ϕ indicate the partial derivatives with respect to semi-toroidal shell coordinates. Therefore, the components of metric tensor at the S are given by

$$E = \bar{r}_\theta\cdot\bar{r}_\theta = r_\theta^2 + Z_\theta^2 = a^2 \tag{9a}$$