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Bending of thin-walled beams of open section with influence of shear, part I: Theory

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ABSTRACT

This two-part contribution presents a novel theory of bending of thin-walled beams with influence of shear (TBTS). The theory is based on the Vlasov's general beam theory as well as on the Timoshenko's beam bending theory. The theory is valid for general thin-walled open beam cross-sections. Part I is devoted to the theoretical developments and part II discusses analytical and obtained numerical results. The theory is based on a kinematics assuming that the cross-section maintains its shape and including three independent warping parameters due to shear. Poison's effect is ignored, as well as warping constrains due to shear (as it known, those effect have small and for engineering praxis neglected influence on the stresses and displacements). Closed-form analytical results are obtained for three-dimensional expressions of the normal and shear stresses. Under general transverse loads, reduced to the cross-section principal pole, the beam will be subjected to bending with influence of shear and in addition to torsion due to shear with respect to the cross-section principal pole and to tension/compression due to shear, in the case of non-symmetrical cross-sections. The beam will be subjected to bending with influence of shear (i) in the plane of symmetry under the loads in that plane and in addition to tension/compression due to shear, (ii) in the plane through the principal pole orthogonal to the plane of symmetry under the loads in that plane and in addition to torsion with respect to the principal pole, in the case of the mono-symmetrical cross-sections. The beam will be subjected to bending with influence of shear in the principal planes, in the case of the bi-symmetrical cross-sections. The principal cross-section axes as well as the principal pole are defined by the classical Vlasov's theory of thin-walled beams of open section. The analytical and numerical analyses presented in part II include comparisons with the classical beam theory, Euler-Bernoulli' theory (EBBT) as well as comparisons with the finite element method (FEM).

1. Introduction

The Euler-Bernoulli's beam theory as well as the Vlasov's thin-walled beam theory [24] do not take into account shear deformations due to shear forces. The shear effect, as well as Poisson's effect, can be included by methods of theory of elasticity [7], but in that case the problem is no longer one-dimensional.

Thus, approximate methods to include the shear effect are developed; especially, in analyses of the beam displacements [19,26], by deriving adequate stiffness matrix [16,17]. For that purpose, the concept of shear factors, first introduced by Timoshenko [20,21], was used. The shear factor was defined by Timoshenko as the ratio of the maximum stress to the average shear stress over a cross section. Recent approaches to the problem are based on geometric assumptions [1,10,12–14,18,23,5,8] or shear energy relations [6,16,17]. Numerical examples comparing results obtained by different approaches can be fined in literatures [2,4], Comparisons to the results of the finite element analysis can be fined in the literature as well [12–14,6].

In this paper, an approximate analytical solution for stresses along the beam cross-section contour as well as solutions for displacements along the beam length will be developed. Various types of transverse loading and boundary condition are assumed. The beams with general cross-section contour shapes and common symmetrical shapes, as special cases, will be investigated.

The Poisson's effect is ignored. Its influence on the stresses as well as displacements in the case of common open thin-walled cross-sections is small, even for extremely low ratios of beam length to cross section contour dimensions [9,17]. The shear warping effect, defined by “non-uniform warping bending theory” [27] will be ignored as well. It is shown that this effect remain very localised close to the clamped ends, where by this theory warping due to shear is restricted. The effect

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of the cross-section distortion will be ignored as well [3,6,11,15,22,25].

2. Stresses and strains

2.1. Strains and displacements

Developed analytical model is based on the following assumptions:

1. The shape of the cross-section middle line is maintained during the beam deformations.
2. The normal stresses can be neglected, except the normal stresses in the longitudinal direction, which are uniformly distributed in the normal direction to the cross-section middle line.
3. The shear stresses can be neglected, except the shear stresses in the direction of the tangent on the cross-section middle line, which are uniformly distributed.

These assumptions are identical to the Vlasov's assumptions of the general thin-walled beam of open section theory, except the Vlasov's major assumption that shear deformation in the beam middle surface can be neglected (against the existing shear stresses in the direction of the tangent on the cross-section middle line).

The following constraints are presumed:

1. The beam is a thin-walled structure with the ratio

$t/d \leq 1/10$

where t is the wall thickness and d is a part of the cross-section middle line (between junctions or between a junction and the free edge). Thus, the thin-walled beam can be considered as a thin-walled structure of a cylindrical or prismatic shell type, with an arbitrary cross-section consisting of the finite number of thin (flat or curved) plates. In Vlasov's beam theory such thin-walled structures are classified as long cylindrical shells, where

$h/l \leq 1/10$

where h is the height (or the width) or the beam cross-section.

Here such a constraint is useless. The theory can be applied even for extremely short thin-walled structures, by assuming that the ratio t/d is small.

2. The cross-section must be stiffened against the cross-section distortion by intermediate diaphragms.

According to the assumption 1. the following expressions for the displacements can be written (Fig. 1):

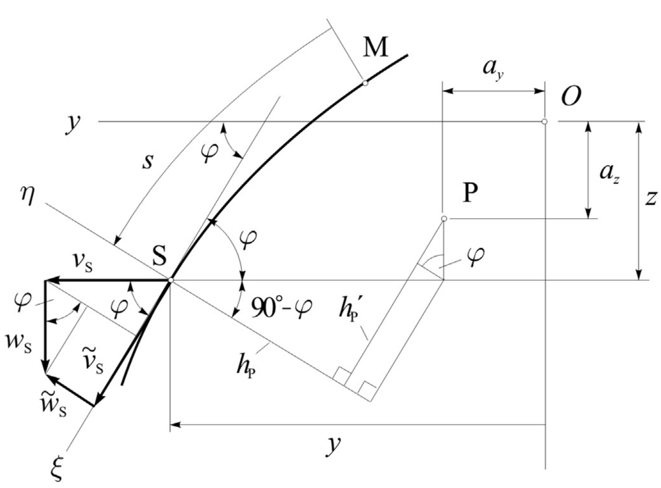

Fig. 1. Displacements of an arbitrary point of the cross-section middle line.

$$v_S = v_P - (z - a_z)\alpha_P,\ \ w_S = w_P + (y - a_y)\alpha_P, \tag{1}$$

.

where $v_S = v_S(x, s)$ and $w_S = w_S(x, s)$ are the displacements of an arbitrary point S of the cross-section contour (the median line) in the y and z-directions, respectively, $y = y(s)$ and $z = z(s)$ are the rectangular coordinates, i.e. the rectangular axes in the cross-section planes, a_y and a_z the coordinates of an arbitrary pole P, s is the curvilinear coordinate of the point S from the starting point M, $v_P = v_P(x)$ and $w_P = w_P(x)$ are the displacements of the pole P in the y and z-directions, respectively, i.e. the displacements of the cross-section contour as rigid contour, and $\alpha_P = \alpha_P(x)$ is the angular displacement of the contour as a rigid contour with respect to the pole P;

$$\tilde{v}_S = v_S \cos\varphi + w_S \sin\varphi, \tag{2}$$

where $\tilde{v}_S = \tilde{v}_S(x, s)$ is the displacement of the point S in the ξ-direction, $Sx'\eta\xi$ is the rectangular coordinate system with respect to the point S, $\varphi = \varphi(s)$ is the angle between the tangent ξ and the axis y and $Oxyz$ is the rectangular coordinate system with respect to the cross-section contour.

Substitution of Eq. (1) into Eq. (2) gives

$$\tilde{v}_S = [\cos\varphi \quad \sin\varphi \quad h_P]\begin{bmatrix} v_P \\ w_P \\ \alpha_P \end{bmatrix}, \tag{3}$$

where

$$h_P = (y - a_y)\sin\varphi - (z - a_z)\cos\varphi, \tag{4}$$

i.e.

$$\tilde{v}_S = \begin{bmatrix} \frac{dy}{ds} & \frac{dz}{ds} & \frac{d\omega}{ds} \end{bmatrix}\begin{bmatrix} v_P \\ w_P \\ \alpha_P \end{bmatrix}, \tag{5}$$

where

$$\cos\varphi = \frac{dy}{ds},\ \ \sin\varphi = \frac{dz}{ds},\ \ h_P = \frac{d\omega}{ds}, \tag{6}$$

$\omega = \omega(s)$ is the sectorial coordinate with respect to the pole P and the point M:

$$\omega = \int_0^s h_P\, ds. \tag{7}$$

The relation between displacements and shear strains in the beam middle surface $\gamma_{x\xi} = \gamma_{x\xi}(x, s)$, can be expressed as

$$\gamma_{x\xi} = \frac{\partial u_S}{\partial s} + \frac{\partial \tilde{v}_S}{\partial x}. \tag{8}$$

Substitution of Eq. (5) into Eq. (8) gives

$$\frac{\partial u_S}{\partial s} = \begin{bmatrix} -\frac{dy}{ds}D & -\frac{dz}{ds}D & -\frac{d\omega}{ds}D \end{bmatrix}\begin{bmatrix} v_P \\ w_P \\ \alpha_P \end{bmatrix} + \gamma_{x\xi}, \tag{9}$$

where $D \equiv \frac{d}{dx}$.

The solution of Eq. (9) can be expressed as follows

$$\begin{bmatrix} u_S^u \\ u_S^v \\ u_S^w \\ u_S^\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -y(s_y)D & 0 & 0 \\ 0 & 0 & -z(s_z)D & 0 \\ 0 & 0 & 0 & -\omega(s)D \end{bmatrix}\begin{bmatrix} u_M \\ v_P \\ w_P \\ \alpha_P \end{bmatrix} + \begin{bmatrix} u_i^u \\ u_j^v \\ u_k^w \\ u_i^\alpha \end{bmatrix} + \begin{bmatrix} \int_0^{s_i} \gamma_{x\xi}^u\, ds \\ \int_0^{s_{yj}} \gamma_{x\xi}^v\, ds_y \\ \int_0^{s_{zk}} \gamma_{x\xi}^w\, ds_z \\ \int_0^{s_i} \gamma_{x\xi}^\alpha\, ds \end{bmatrix}, \tag{10}$$

where