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Bending of thin-walled beams of open section with influence of shear—Part II: Application

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ABSTRACT

This two-part contribution presents a novel theory of bending of thin-walled beams with influence of shear (TBTS). The theory is valid for general open thin-walled cross-sections. The loads are reduced to the cross-section principal pole, i.e. the cross-section shear center. In part I, the TBTS is established, in this part II, the theory is used to analyze various beam cross-section shapes, loads and boundary conditions. Beams with the extreme low ratios of beam length to beam cross-section contour dimensions are analyzed. The stress predictions of the TBTS, stresses as functions of the longitudinal coordinate as well as the cross-section curvilinear coordinate, are compared to those obtained by finite elements computations (FEM) as well as to exact solutions of the theory of elasticity. The results of the TBTS are given in closed analytic forms, i.e. parametric forms, suitable for general studies of thin-walled beam behavior under transvers bending loads, as well in the early design stage of thin-walled structures.

1. Introduction

The theory of bending of thin-walled beams with influence of shear (TBTS) established in part I [10] is based on the classical theories of thin-walled beams of open cross-sections [15] improved by the influence of shear, i.e. the cross-section warping due to shear [4,9,10,12,13,14,16]. The cross-section contour distortion is ignored [1,3,11], as well as Poisson's effect [2].

To illustrate the TBTS, this part II is devoted to analytical and numerical analyzes of simple supported and clamped beams, subjected to uniform transverse bending loads acting through the cross-section principal pole. The stresses and displacements are analyzed for non-symmetric thin-walled cross-sections, mono-symmetrical and bi-symmetrical cross-sections.

In the case of non-symmetric cross-sections, the beam will be subjected to bending with influence of shear and in addition to tension (compression) and torsion due to shear; in the case of mono-symmetric cross-sections with the transverse loads in the plane of symmetry, the beam will be subjected to bending with influence of shear in that plane and in addition to tension (compression); in the case of loads through the principal pole, orthogonal to the plane of symmetry, the beam will be subjected to bending with influence of shear in that plane an in addition to torsion due to shear. In the case of bi-symmetric cross-section with load through the principal pole, the beam will be subjected to bending with influence of shear only.

The obtained results will be compared to the results of the finite element calculations as well as to results of some simple examples of the theory of elasticity. Some results of from available literatures will be discussed also.

The symbols used in this paper are those specified in the part I.

2. The theory

2.1. General case: bending with influence of shear with additional tension and torsion due to shear

In the case of a non-symmetric open thin-walled cross-section, with the transverse loads reduced to the components q_y and q_z through the cross-section principal pole P, the normal stresses can be expressed as function of primary internal force components $M_y = M_y(x)$, $M_z = M_z(x)$ and secondary internal force components $N^y = N^y(x)$, $N^z = N^z(x)$, M_y^y, $M_y^z(x)$, $M_z^y(x)$, $M_z^z(x)$, $B^y(x)$ and $B^z(x)$:

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$$\begin{bmatrix}\sigma_x^u\\ \sigma_x^v\\ \sigma_x^w\\ \sigma_x^\alpha\end{bmatrix}=\begin{bmatrix}1&0&0&0\\0&y&0&0\\0&0&z&0\\0&0&0&\omega\end{bmatrix}\left(\begin{bmatrix}0\\-\frac{M_z}{I_z}\\ \frac{M_y}{I_y}\\0\end{bmatrix}+\begin{bmatrix}\frac{N^y}{A}+\frac{N^z}{A}\\-\frac{M_z^y}{I_z}-\frac{M_z^z}{I_z}\\ \frac{M_y^y}{I_y}+\frac{M_y^z}{I_y}\\ \frac{B^y}{I_\omega}+\frac{B^z}{I_\omega}\end{bmatrix}\right)-\frac{E}{G}\begin{bmatrix}0\\ \sigma_{xj}^v+\frac{q_y}{I_z}\int_0^{s_{yj}}\frac{S_z^*}{t}\mathrm{d}s\\ \sigma_{xk}^w+\frac{q_z}{I_y}\int_0^{s_{zk}}\frac{S_y^*}{t}\mathrm{d}s\\0\end{bmatrix},\ \sigma_x=\sigma_x^u+\sigma_x^v+\sigma_x^w+\sigma_x^\alpha. \tag{1}$$

where

$$\begin{bmatrix}N^y\\M_z^y\\M_y^y\\B^y\end{bmatrix}=\frac{E}{G}q_y\begin{bmatrix}L_a\kappa_{xy}\\-\frac{I_z}{A}\kappa_{yy}\\ \frac{I_y}{A}\kappa_{zy}\\ \frac{I_\omega}{W_P}\kappa_{\omega y}\end{bmatrix},\quad \begin{bmatrix}N^z\\M_z^z\\M_y^z\\B^z\end{bmatrix}=\frac{E}{G}q_z\begin{bmatrix}L_a\kappa_{xz}\\-\frac{I_z}{A}\kappa_{yz}\\ \frac{I_y}{A}\kappa_{zz}\\ \frac{I_\omega}{W_P}\kappa_{\omega z}\end{bmatrix}, \tag{2}$$

where

$$\kappa_{xy}=\frac{1}{I_zL_a}\int_A\frac{A_y^*S_z^*}{t^2}\mathrm{d}A,\quad \kappa_{xz}=\frac{1}{I_yL_a}\int_A\frac{A_z^*S_y^*}{t^2}\mathrm{d}A,$$
$$\kappa_{yy}=\frac{A}{I_z^2}\int_A\left(\frac{S_z^*}{t}\right)^2\mathrm{d}A,\ \kappa_{yz}=\kappa_{zy}=\frac{A}{I_yI_z}\int_L\frac{S_y^*S_z^*}{t}\mathrm{d}s,$$
$$\kappa_{zz}=\frac{A}{I_y^2}\int_A\left(\frac{S_y^*}{t}\right)^2\mathrm{d}A,\quad \kappa_{\omega y}=\frac{W_P}{I_zI_\omega}\int_L\frac{S_z^*S_\omega^*}{t}\mathrm{d}s,\ \kappa_{\omega z}$$
$$=\frac{W_P}{I_yI_\omega}\int_L\frac{S_y^*S_\omega^*}{t}\mathrm{d}s \tag{3}$$

are the shear factors;

$$A^*=\int_{s^*}\mathrm{d}A^*,\ S_z^*=\int_{s_y^*}y\ \mathrm{d}A^*,\ S_y^*=\int_{s_z^*}z\ \mathrm{d}A^*,\ S_\omega^*=\int_{s^*}\omega\ \mathrm{d}A^*,$$

are the properties of the cut-off portions of cross-section area.

The stress components σ_{xj}^v and σ_{xk}^w can be obtained from the compatibility conditions

$$\sigma_{xj}^v=\sigma_{xj+1}^v(\varepsilon_{xj}^v=\varepsilon_{x,j+1}^v),\ \sigma_{xj}^v+q_y\frac{E}{GI_z}\int_0^{s_{yj}}\frac{S_z^*}{t}\mathrm{d}s\Big|_{s_{yj}=L_{yj}}$$
$$=\sigma_{x,j+1}^v+q_y\frac{E}{GI_z}\int_0^{s_{y,j+1}}\frac{S_z^*}{t}\mathrm{d}s\Big|_{s_y=-L_{y,j+1}};\ \sigma_{xk}^w=\sigma_{x,k+1}^w(\varepsilon_{xk}^w=\varepsilon_{x,k+1}^w),\ \sigma_{xk}^w$$
$$+q_z\frac{E}{GI_y}\int_0^{s_{zk}}\frac{S_y^*}{t}\mathrm{d}s\Big|_{s_{zk}=L_{zk}}=\sigma_{x,k+1}^w+q_z\frac{E}{GI_y}\int_0^{s_{z,k+1}}\frac{S_y^*}{t}\mathrm{d}s\Big|_{s_z=-L_{z,k+1}}, \tag{4}$$

where

$$0\le s_{yj}\le L_{yj}^+,\ 0\le s_{yj}^*\le L_{yj}^+,\ 0\ge s_{y,j+1}\ge -L_{y,j+1}^-,\ 0\ge s_{y,j+1}^*\ge -L_{y,j+1}^-,\ 0$$
$$\le s_{zk}\le L_{zk}^+,\ 0\le s_{zk}^*\le L_{zk}^+,\ 0\ge s_{z,k+1}\ge -L_{z,k+1}^-,\ 0\ge s_{z,k+1}^*\ge -L_{z,k+1}^-, \tag{5}$$

L_{yj} and $L_{y,j+1}$ are the distances from the starting points, where $z=0$, to the free edges, where $S_z^*=0$, i.e. to arbitrary points between starting points, both L_{zk} and $L_{z,k+1}$ are the distances of the staring points, where $y=0$, to free edges, where $S_y^*=0$. i.e. to arbitrary points between starting points.

The normal stresses can also be expressed as:

$$\begin{bmatrix}\sigma_x^u\\ \sigma_x^v\\ \sigma_x^w\\ \sigma_x^\alpha\end{bmatrix}=\begin{bmatrix}1&0&0&0\\0&y&0&0\\0&0&z&0\\0&0&0&\omega\end{bmatrix}\left(\begin{bmatrix}0\\-\frac{M_z}{I_z}\\ \frac{M_y}{I_y}\\0\end{bmatrix}+\frac{E}{G}\begin{bmatrix}\frac{L_a}{A}\kappa_{xy}&\frac{L_a}{A}\kappa_{xz}\\ \frac{\kappa_{yy}}{A}&\frac{\kappa_{yz}}{A}\\ \frac{\kappa_{zy}}{A}&\frac{\kappa_{zz}}{A}\\ \frac{\kappa_{\omega y}}{W_P}&\frac{\kappa_{\omega z}}{W_P}\end{bmatrix}\begin{bmatrix}q_y\\q_z\end{bmatrix}\right)-\frac{E}{G}\begin{bmatrix}0\\ \sigma_{xj}^v+\frac{q_y}{I_z}\int_0^{s_{yj}}\frac{S_z^*}{t}\mathrm{d}s\\ \sigma_{xk}^w+\frac{q_z}{I_y}\int_0^{s_{zk}}\frac{S_y^*}{t}\mathrm{d}s\\0\end{bmatrix}$$

or

$$\begin{bmatrix}\sigma_x^u\\ \sigma_x^v\\ \sigma_x^w\\ \sigma_x^\alpha\end{bmatrix}=\begin{bmatrix}1&0&0&0\\0&y&0&0\\0&0&z&0\\0&0&0&\omega\end{bmatrix}\left(\begin{bmatrix}0\\-\frac{M_z}{I_z}\\ \frac{M_y}{I_y}\\0\end{bmatrix}+\frac{E}{G}\begin{bmatrix}\frac{L_a}{A}\kappa_{xy}&\frac{L_a}{A}\kappa_{xz}\\ \frac{1}{A_{yy}}&\frac{1}{A_{yz}}\\ \frac{1}{A_{zy}}&\frac{1}{A_{zz}}\\ \frac{1}{W_{Py}}&\frac{1}{W_{Pz}}\end{bmatrix}\begin{bmatrix}q_y\\q_z\end{bmatrix}\right)-\frac{E}{G}\begin{bmatrix}0\\ \sigma_{xj}^v+\frac{q_y}{I_z}\int_0^{s_{yj}}\frac{S_z^*}{t}\mathrm{d}s\\ \sigma_{xk}^w+\frac{q_z}{I_y}\int_0^{s_{zk}}\frac{S_y^*}{t}\mathrm{d}s\\0\end{bmatrix} \tag{6}$$

where

$$\begin{bmatrix}A_{yy}&A_{yz}\\A_{zy}&A_{zz}\\W_{Py}&W_{Pz}\end{bmatrix}=\begin{bmatrix}\frac{A}{\kappa_{yy}}&\frac{A}{\kappa_{yz}}\\ \frac{A}{\kappa_{zy}}&\frac{A}{\kappa_{zz}}\\ \frac{W_P}{\kappa_{\omega y}}&\frac{W_P}{\kappa_{\omega z}}\end{bmatrix}. \tag{7}$$

The shear stresses can be expressed as

$$\begin{bmatrix}\tau_{x\xi}^u\\ \tau_{x\xi}^v\\ \tau_{x\xi}^w\\ \tau_{x\xi}^\alpha\end{bmatrix}=\frac{1}{t}\begin{bmatrix}A^*&0&0&0\\0&S_z^*&0&0\\0&0&S_y^*&0\\0&0&0&S_\omega^*\end{bmatrix}\left(\begin{bmatrix}0\\-\frac{Q_y}{I_z}\\ \frac{Q_z}{I_y}\\0\end{bmatrix}+\frac{E}{G}\begin{bmatrix}\frac{L_a}{A}\kappa_{xy}&\frac{L_a}{A}\kappa_{xz}\\ \frac{\kappa_{yy}}{A}&\frac{\kappa_{yz}}{A}\\ \frac{\kappa_{zy}}{A}&\frac{\kappa_{zz}}{A}\\ \frac{\kappa_{\omega y}}{W_P}&\frac{\kappa_{\omega z}}{W_P}\end{bmatrix}\mathrm{D}\begin{bmatrix}q_y\\q_z\end{bmatrix}+\frac{E}{G}\cdot\frac{\partial}{\partial x}\begin{bmatrix}0\\ \int_{s_{yj}^*}\left(\sigma_{xj}^v+\frac{q_y}{I_z}\int_{s_{yj}^*}\frac{S_z^*}{t}\mathrm{d}s\right)\mathrm{d}s\\ \int_{s_{zk}^*}\left(\sigma_{xk}^v+\frac{q_z}{I_y}\int_{s_{zk}^*}\frac{S_y^*}{t}\mathrm{d}s\right)\mathrm{d}s\\0\end{bmatrix}\right),$$

or

$$\begin{bmatrix}\tau_{x\xi}^u\\ \tau_{x\xi}^v\\ \tau_{x\xi}^w\\ \tau_{x\xi}^\alpha\end{bmatrix}=\frac{1}{t}\begin{bmatrix}A^*&0&0&0\\0&S_z^*&0&0\\0&0&S_y^*&0\\0&0&0&S_\omega^*\end{bmatrix}\left(\begin{bmatrix}0\\-\frac{Q_y}{I_z}\\ \frac{Q_z}{I_y}\\0\end{bmatrix}+\frac{E}{G}\begin{bmatrix}\frac{L_a}{A}\kappa_{xy}&\frac{L_a}{A}\kappa_{xz}\\ \frac{1}{A_{yy}}&\frac{1}{A_{yz}}\\ \frac{1}{A_{zy}}&\frac{1}{A_{zz}}\\ \frac{1}{W_{Py}}&\frac{1}{W_{Pz}}\end{bmatrix}\mathrm{D}\begin{bmatrix}q_y\\q_z\end{bmatrix}+\frac{E}{G}\cdot\frac{\partial}{\partial x}\begin{bmatrix}0\\ \int_{s_{yj}^*}\left(\sigma_{xj}^v+\frac{q_y}{I_z}\int_{s_{yj}^*}\frac{S_z^*}{t}\mathrm{d}s\right)\mathrm{d}s\\ \int_{s_{zk}^*}\left(\sigma_{xk}^v+\frac{q_z}{I_y}\int_{s_{zk}^*}\frac{S_y^*}{t}\mathrm{d}s\right)\mathrm{d}s\\0\end{bmatrix}\right), \tag{8}$$

where