Thin-Walled Structures 114 (2017) 22–31

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Full length article

# Dynamic analysis of high-speed railway bridge decks using generalised beam theory

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ARTICLE INFO

*Keywords:*
Generalised Beam Theory (GBT)
High-speed railway bridges
Local vibrations
Distortional vibrations
Resonance
Dynamic response

ABSTRACT

This paper presents the details and illustrates the application of a semi-analytical Generalised Beam Theory (GBT) formulation for the dynamic analysis of high-speed railway bridge decks. The technique, which takes into account the cross-section local/distortional and shear deformations, provides an original "doubly modal" representation of the dynamic response, which makes it possible to acquire deep insight into the mechanics of the deck structural behaviour. It is employed to investigate the dynamic behaviour of a real high-speed railway viaduct, comprising a series of identical 46*m*-span simply supported box girder pre-stressed concrete decks carrying two tracks – the augmented structural response associated with resonance due to the crossing of trains at high-speeds is captured. The results obtained provide evidence that local/distortional deformation can play a significant role on the dynamic response of such structures. Moreover, the GBT analyses, whose validity is confirmed through the comparison with values yielded by shell finite element analyses, are shown to provide accurate results while involving just a few degrees of freedom.

## 1. Introduction

High-speed railway has proved an environmentally efficient competitor with air or road transport, with many countries currently building or expanding their networks (*e.g.*, [1]). However, the state-of-the-art technology involved is associated with many specific and complex engineering problems, such as the need to estimate/mitigate the vibrations induced by the crossing of high-speed trains on the track, ground, bridge decks, tunnels, nearby buildings, *etc.* Concerning bridge decks, previous investigations have shown that the crossing of a train (modelled as a set of "equally spaced" moving loads) travelling at high speed constitutes an action that might be in *resonance* with the deck natural vibration frequencies/modes (*e.g.*, [2–7]). When this happens, the ensuing vibrations are bound to have a strong impact on the bridge (i) structural safety, (ii) track stability and wheel-rail contact, and also (iii) passenger comfort. In order to avoid these problems, the design of such bridges may require the performance of a dynamic analysis, as specified in Eurocode 2 (EN1991-2 [8]). Usually, such analysis assumes that the deck response involves only *global* deformations, namely flexural and/or torsional displacements. However, this assumption may not be adequate in the case of slender thin-walled decks, where it may happen that *local* deformation (*i.e.*, deformation modes causing a change in the cross-section shape, *e.g.*, involving transverse bending curvatures in its walls) plays an important role in their dynamic response [9,10].

In order to perform local/global dynamic analyses of thin-walled prismatic members, such as many bridge decks, two main types of numerical analysis are usually employed: (i) shell finite element analyses [11] or (ii) finite strip analyses [12]. The more recent Generalised Beam Theory (GBT) is an alternative approach, which has not yet been applied in this specific context (*e.g.*, [13,14]). The trademark of GBT is the possibility of discretising the member deformed configuration (*e.g.*, a vibration mode shape) by means of a linear combination of structurally meaningful *cross-section deformation modes*, whose amplitudes (the problem unknowns) generally vary along length and time dimensions. This unique feature makes it possible (i) to acquire a better understanding about the mechanics of the member dynamics and also (ii) to perform very efficient analyses, in the sense that the number of degrees of freedom involved is fairly low. In recent years, GBT has been employed in the context of free-vibration analysis of beams and structural systems (*e.g.*, [15–19]). Concerning the *dynamic analysis* of thin-walled members, it is worth noting the original "doubly modal" representation of the response obtained if the GBT approach is combined with the traditional vibration mode superposition approach

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http://dx.doi.org/10.1016/j.tws.2017.01.027
Received 2 November 2016; Received in revised form 28 December 2016; Accepted 24 January 2017

[20,21]. This makes it possible to analyse/decompose the response in terms of contributions from (i) deformation modes and/or (ii) vibration modes. A detailed account of the development of the above GBT formulation for dynamic analysis can be found in [20]. It also includes numerical results concerning a few simple classical problems of structural dynamics, aimed at illustrating the application, capabilities and potential of the approach. Therefore, only a brief overview of the GBT formulation employed in this work is presented here (in Section 2).

The application of the GBT formulation for dynamic analysis "goes one step further" in this work, which explores its use to analyse a complex "real-life engineering problem", involving the deck of an existing high-speed railway bridge. This bridge deck was previously analysed by means of several structural models exhibiting different levels of complexity (simple beam, grid and shell finite elements) [10]. It is shown here that the GBT approach unique doubly-modal features and numerical efficiency make it possible to unveil key aspects of the bridge deck structural behaviour with a fairly small computational cost (accurate results, including local deformations, are achieved without resorting to computer-intensive shell finite element models).

A structural model able to simulate adequately the dynamic response of a real railway bridge deck, when crossed by high-speed trains, is developed and analysed thoroughly by means of GBT. It consists of a simply supported pre-stressed concrete box girder, which is part of the El Genil viaduct currently in service in the Spanish Córdoba-Málaga high-speed line [10]. The development of the above model, on the basis of the real bridge deck geometry and material properties, is far from trivial, since it is necessary to look for simplicity without sacrificing the accuracy of the results. The maximum deck vertical displacements and accelerations are obtained, as a function of the train crossing speed, for the crossing of (i) the 10 representative train configurations comprising the "High Speed Load Model A" of Eurocode 1 (HSLM-A) [8] and (ii) 7 real European high-speed trains [3]. Particular attention is paid to the influence of local deformation on the deck vibration and dynamic behaviour. The GBT-based results are validated by means of the comparison with values yielded by shell finite element analyses performed in the commercial code SAP2000 [22]. It is shown that GBT constitutes a viable alternative to more sophisticated models (*e.g.*, shell finite element ones) in unveiling the behaviour of this complex real-life thin-walled member problem.

## 2. GBT-based dynamic analysis – overview

Since the cross-section displacement field is expressed as a linear combination of structurally meaningful deformation modes, a GBT-based dynamic analysis (i) leads to equilibrium equations written in a rather convenient form and, as mentioned before, (ii) makes it possible to perform "doubly modal" analyses that provide in-depth and fresh insight on the mechanics of the dynamic behaviour of prismatic thin-walled members.

### 2.1. Basic formulation

In order to derive the GBT equilibrium equations, consider the prismatic thin-walled member depicted in Fig. 1(a), with a supposedly arbitrary open cross-section – also shown is the member global coordinate system $X-Y-Z$ (longitudinal, major and minor axis). Moreover, local coordinate systems $x-s-z$ are adopted in each wall, as shown in Fig. 1(b), where (i) $x$ (parallel to $X$) and $s$ define the wall mid-plane and (ii) $z$ is measured along the thickness $h$. In this coordinate system, the mid-line displacement field components are $u$, $v$ and $w$.

By employing Kirchhoff's hypothesis, the displacement vector of an arbitrary member point $\mathbf{U}(x, s, z, t)$ is expressed as

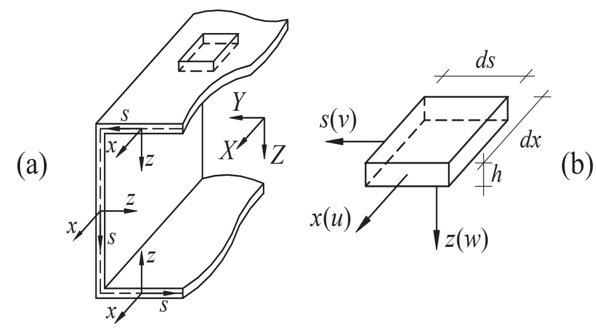


**Fig. 1.** (a) Arbitrary prismatic open-section thin-walled member and global coordinate system, and (b) wall element with its local coordinate system and displacement components.

$$\mathbf{U}=\begin{Bmatrix} U_x \\ U_s \\ U_z \end{Bmatrix}=\begin{Bmatrix} u-zw_{,x} \\ v-zw_{,s} \\ w \end{Bmatrix}, \tag{1}$$

where $(\bullet)_{,x}=\partial(\bullet)/\partial x$. In accordance with the classical thin-walled bar theory, the mid-plane displacement field can be conveniently expressed as (the summation convention applies to subscript $k$)

$$u(x, s, t)=u_k(s)\zeta_{k,x}(x, t), \tag{2a}$$

$$v(x, s, t)=v_k(s)\zeta_k(x, t), \tag{2b}$$

$$w(x, s, t)=w_k(s)\zeta_k(x, t), \tag{2c}$$

where (i) $u_k(s)$, $v_k(s)$ and $w_k(s)$ are the mid-line functions defining cross-section deformation mode $k$ (or "GBT mode $k$"), (ii) $\zeta_k(x, t)$ or $\zeta_{k,x}(x, t)$ are the amplitude functions describing their variation along the member length and time, and (iii) $1 \le k \le N_d$, where $N_d$ is the total number of deformation modes included in the analysis. Therefore, the member deformed configuration can be expressed as a sum of contributions from the $N_d$ deformation modes. Eqs. (2a-2c) can be conveniently written as inner products of $N_d$-dimension vectors,

$$u = \mathbf{u}^T\boldsymbol{\zeta}_{,x} \qquad v = \mathbf{v}^T\boldsymbol{\zeta} \qquad w = \mathbf{w}^T\boldsymbol{\zeta}, \tag{3}$$

where $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ contain the $u_k(s)$, $v_k(s)$ and $w_k(s)$ functions, respectively, and $\boldsymbol{\zeta}$ contains the corresponding amplitude functions $\zeta_k(x, t)$.

In order to perform dynamic analyses of thin-walled members, if one applies Hamilton's Principle, neglecting the geometrically non-linear effects of the applied loading,[1] it leads to the member dynamic equilibrium equations and reads

$$\int_{t_1}^{t_2} \delta(U+\Pi-T)dt=0, \tag{4}$$

where (i) $U$ and $T$ are the member total strain and kinetic energies, (ii) $\Pi$ is the potential of the applied loads, (iii) $t_1$ and $t_2$ are the initial and final time instants and (iv) the dissipative forces are deemed null. The expressions providing $U$, $T$ and $\Pi$ can be found in [20].

Assuming a St. Venant-Kirchhoff material behaviour, small displacements and a plane stress state [20]; carrying out the time integration in Eq. (4) leads to the variational (weak) form of the equilibrium equations,[2]

$$\int_L \begin{pmatrix} \delta\boldsymbol{\zeta}^T_{,xx}\mathbf{C}\boldsymbol{\zeta}_{,xx}+\delta\boldsymbol{\zeta}^T_{,x}\mathbf{D}\boldsymbol{\zeta}_{,x}+\delta\boldsymbol{\zeta}^T B\boldsymbol{\zeta}+\delta\boldsymbol{\zeta}^T_{,xx}E\boldsymbol{\zeta}+\delta\boldsymbol{\zeta}^T\mathbf{E}^T\boldsymbol{\zeta}_{,xx}+ \\ +\,\delta\boldsymbol{\zeta}^T_{,x}\mathbf{Q}\boldsymbol{\zeta}_{,xtt}+\delta\boldsymbol{\zeta}^T\mathbf{R}\boldsymbol{\zeta}_{,tt}-\delta\boldsymbol{\zeta}^T\mathbf{q}\varphi \end{pmatrix}dx=0, \tag{5}$$

where (i) $\mathbf{C}$, $\mathbf{B}$, $\mathbf{D}$ and $\mathbf{E}$ are linear stiffness matrices, associated with primary/secondary warping, transverse extension/flexure, membrane/

[1] For a thorough discussion on geometrically non-linear effects of applied loading, see *e.g.* [15,16].

[2] The ensuing boundary condition term is deemed null by considering of $\delta\boldsymbol{\zeta}(x,t_1)=\delta\boldsymbol{\zeta}(x,t_2)=0$.