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Vibration of thermally post-buckled hybrid laminates with two non-uniformly distributed fibers

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ABSTRACT

This study presents the first vibration analysis of thermally post-buckled hybrid laminates with non-uniformly distributed graphite and E-glass fibers in a single lamina. A 54 degree-of-freedom high order triangular plate element is developed based on the Von Karman large deflection assumption. The formulation of the location dependent linear, nonlinear stiffness and mass matrices due to non-homogeneous material properties, geometrical stiffness matrix due to thermal effects and thermal moment vector due to temperature gradient is derived. The effects of hybrid-fiber distribution on the natural frequencies and mode shapes of thermally post-buckled laminates are discussed. The numerical results reveal that the redistribution of two fibers can considerably decrease the postbuckling deflections of the hybrid laminates and significantly modify the natural frequencies. The stiffening effect of fiber redistribution is more obvious for the laminate with a higher volume fraction index and clamped edges. Special buckling and vibration mode shapes are observed, along with multiple vibration mode shifting.

1. Introduction

Different fibers are often mixed in the textile industry. However, the application of hybrid fiber composites is rare in other fields, and few studies report the structural behaviors of the laminates with different fibers [1–5]. But, the fibers in each lamina are still the same for graphite-kevlar-graphite plates [1], carbon/epoxy and glass/epoxy laminates [2,3], hybrid laminated composites with a hemp natural fiber/polypropylene core and two glass fiber/polypropylene surface layers [4], unidirectional glass fiber/random glass fiber/epoxy hybrid composites [5].

Recently, Kuo [6] proposed that two different fibers may be non-uniformly distributed in a single lamina and tailored to achieve the specific requirements of structural strength and lower costs. This is particularly useful for thin-walled structures with aircraft and space applications. Although the laminates with variable fiber spacing [7–12] may avoid stress concentrations, the fiber volume fraction may be quite low at the edges of the laminate. It is more practical to replace some graphite fibers with E-glass fibers instead of epoxy matrix in the outer portion of the laminate. The numerical results reveal that the redistribution of two fibers can efficiently increase the critical buckling temperature, natural frequencies and flutter boundary [6]. These earlier works thus shed new light on the potential development of a "smarter layout" for such composites.

The plates can carry an additional load after buckling without failure. In other words, the plates can be used at temperatures higher than the critical buckling temperature. Many studies have examined the thermal postbuckling of composite laminates [13–16] and the vibration of buckled composite laminates [17–21]. Raju and Rao [13] investigated the thermal postbuckling behavior of orthotropic square plates with simply supported edges using the Rayleigh-Ritz method. Chen and Chen [14] studied the thermal postbuckling behavior of laminates subjected to a uniform thermal loading. Meyers and Hyer [15] revealed that support conditions and skewing of the material axis may influence modal interaction. Singh et al. [16] showed that the postbuckling path may not remain stable throughout the process.

A plate in its buckled state may survive under dynamic disturbances. Yang and Han [17] investigated the vibration behavior of a simply supported rectangular postbuckled plate using the finite element method. Many triangular elements can be used for plane stress and plate bending problems. Yang and Han [17] proposed that a high-order triangular membrane element could be combined with a fully conforming triangular plate bending element to solve the geometrically nonlinear problems of plates. The results indicate that the T54 high precision triangular plate element is capable of accurately solving a wide variety of geometrically nonlinear plate problems, such as large deflections of the plate, postbuckling of the plate, and free vibration of the plate with in-plane stresses.

Zhou et al. [18] showed that the maximum thermal postbuckling deflection is not always at the center of the plate, and the lowest

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frequency of vibration about the buckled position may not be the (1, 1) mode. Locke [19] evaluated the effects of heating on the vibration frequencies and mode shapes of a single-layer boron/epoxy laminate with free edge conditions. Librescu et al. [20] presented the vibration response of transversely isotropic flat and curved panels subjected to temperature fields and to mechanical loads in the postbuckling load range. Oh and Lee [21] presented the vibration mode shapes and thermal buckling deflection of cross-ply laminated panels. The results showed that thermal snapping changes the vibration characteristics and static deformations.

The buckling and vibration behaviors of the laminates embedded with shape memory alloys [22–25] or piezoelectric actuators [26–28], or made of functionally graded material [29–31] have been examined by many researchers. However, none of the published literature reports the effects of hybrid fibers on the vibration of thermally buckled hybrid laminates with two non-uniformly distributed fibers, although this may reinforce the thin plate more directly without changing the geometry or adding other components. Could a "smarter layout" be achieved by non-uniformly distributing relatively stronger fibers in the required region of a single lamina? What is the dynamic behavior of a hybrid laminate in the thermally post-buckled region? This study aims to provide the answers to these questions based on the successful experience of a previous work [6] which first presented the thermal buckling, vibration and flutter analyses of composite laminates containing two non-uniformly distributed fibers in a single lamina.

In this study, a 54 degree-of-freedom high precision triangular plate element is developed based on the Von Karman large deflection assumption. The formulation of the location dependent linear stiffness matrix, first-order nonlinear stiffness matrix, second-order nonlinear stiffness matrix, geometrical stiffness matrix due to thermal effects and thermal moment vector due to temperature gradient is derived. The effects of the hybrid-fiber distribution on the natural frequencies and mode shapes of thermally post-buckled laminates are discussed in detail.

2. Mathematical formulation

Consider a rectangular hybrid laminate with length a, width b, and thickness h. The laminate is assumed to consist of N layers of orthotropic sheets bonded together. Two different fibers in each lamina are aligned parallel to the longitudinal direction, but distributed unevenly in the transverse direction. Therefore, these two fiber volume fractions, V_{f1} and V_{f2}, are functions of a nondimensional coordinate $\zeta = \frac{2y}{b}$ having its origin at the center of the plate. For the fiber distribution functions [6]

$$V_{f1}(\zeta) = ((V_f)_{in} - (V_f)_{out})(1 - \zeta^2)^n + (V_f)_{out},\ \ n = 1,\ 2,\ 3 \tag{1a}$$

$$V_{f2}(\zeta) = 1 - V_m(\zeta) - V_{f1}(\zeta) \tag{1b}$$

where $(V_f)_{in}$ is the fiber volume fraction at the center of the plate ($\zeta = 0$) and $(V_f)_{out}$ is the fiber volume fraction at the edges ($\zeta = \pm 1$), the volume fraction index n controls the variation of the volume fraction, $V_m(\zeta)$ is the matrix volume fraction. It is seen that the more first fibers are centralized in the central portion of the laminate with a higher volume fraction $(V_f)_{in}$ and higher volume fraction index n. With the fiber distribution functions, the elastic moduli E_1, E_2, ν_{12}, G_{12}, mass density ρ, and the coefficients of thermal expansion α_1 and α_2 for the composite material are also functions of ζ. The formulas used for the calculation of the effective engineering constants are based on the following extended rules of mixture.

$$E_1(\zeta) = E_{f1}V_{f1}(\zeta) + E_{f2}V_{f2}(\zeta) + E_mV_m(\zeta) \tag{2a}$$

$$\frac{1}{E_2(\zeta)} = \frac{V_{f1}(\zeta)}{E_{f1_1}} + \frac{V_{f2}(\zeta)}{E_{f2}} + \frac{V_m(\zeta)}{E_m} \tag{2b}$$

$$\nu_{12}(\zeta) = \nu_{f1}V_{f1}(\zeta) + \nu_{f2}V_{f2}(\zeta) + \nu_mV_m(\zeta) \tag{2c}$$

$$\frac{1}{G_{12}(\zeta)} = \frac{V_{f1}(\zeta)}{G_{f1}} + \frac{V_{f2}(\zeta)}{G_{f2}} + \frac{V_m(\zeta)}{G_m} \tag{2d}$$

$$\rho(\zeta) = \rho_{f1}V_{f1}(\zeta) + \rho_{f2}V_{f2}(\zeta) + \rho_mV_m(\zeta) \tag{2e}$$

$$\alpha_1(\zeta)E_1(\zeta) = \alpha_{f1}E_{f1}V_{f1}(\zeta) + \alpha_{f2}E_{f2}V_{f2}(\zeta) + \alpha_mE_mV_m(\zeta) \tag{2f}$$

$$\alpha_2(\zeta) + \nu_{12}(\zeta)\alpha_1(\zeta) = (1 + \nu_{f1})\alpha_{f1}V_{f1}(\zeta) + (1 + \nu_{f2})\alpha_{f2}V_{f2}(\zeta) + (1 + \nu_m)\alpha_mV_m(\zeta) \tag{2g}$$

The temperature distribution $\Delta T(z)$ may vary across the panel thickness as

$$\Delta T(z) = \frac{T_t + T_b}{2} + \frac{T_t - T_b}{h}z = T_u + \frac{T_g}{h}z \tag{3}$$

where the temperature rise on the top surface (T_t) is assumed to be higher than that on the bottom surface (T_b) if the temperature gradient (T_g) is considered. The displacement field may be expressed as

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \begin{Bmatrix} u_0 \\ v_0 \\ w_0 \end{Bmatrix} - z\begin{Bmatrix} w_{0,x} \\ w_{0,y} \\ 0 \end{Bmatrix} \tag{4}$$

where u_0, v_0 and w_0 are the in-plane and out-of-plane displacement components in the mid-plane of the laminate, respectively. Adopting the Von Karman model accounts for moderately large deflections, the kinematic relation can be determined as

$$\{\varepsilon\} = \{\varepsilon_0\} + z\{\kappa\} + \{\delta\} \tag{5}$$

where $\{\varepsilon_0\}$, $\{\kappa\}$, and $\{\delta\}$ are the strain, plate curvature, and large deflection strain in the mid-plane of the laminate. The location dependent stress-strain relation of the hybrid laminate subjected to the temperature rise ΔT is given by

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11}(\zeta) & Q_{12}(\zeta) & 0 \\ Q_{12}(\zeta) & Q_{22}(\zeta) & 0 \\ 0 & 0 & Q_{66}(\zeta) \end{bmatrix}\left(\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} - \Delta T\begin{Bmatrix} \alpha_1(\zeta) \\ \alpha_2(\zeta) \\ 0 \end{Bmatrix}\right) \tag{6}$$

where

$$Q_{11}(\zeta) = \frac{E_1(\zeta)}{1 - \nu_{12}(\zeta)\nu_{21}(\zeta)} \tag{7a}$$

$$Q_{12}(\zeta) = \frac{\nu_{21}(\zeta)E_1(\zeta)}{1 - \nu_{12}(\zeta)\nu_{21}(\zeta)} \tag{7b}$$

$$Q_{22}(\zeta) = \frac{E_2(\zeta)}{1 - \nu_{12}(\zeta)\nu_{21}(\zeta)} \tag{7c}$$

$$Q_{66}(\zeta) = G_{12}(\zeta) \tag{7d}$$

The stress-strain relation of the k^{th} layer of the hybrid laminate can be expressed as

$$\{\sigma\}_k = [\overline{Q}]_k(\{\varepsilon\} - \Delta T\{\alpha\}_k) \tag{8}$$

where $\overline{Q}_{ij}$ are the transformed reduced stiffness. The force and moment resultants of the hybrid symmetrically laminated plate are defined as

$$\begin{bmatrix} \{N\} \\ \{M\} \end{bmatrix} = \begin{bmatrix} [A] & [0] \\ [0] & [D] \end{bmatrix}\left(\begin{bmatrix} \{\varepsilon^0\} + \{\delta\} \\ \{\kappa\} \end{bmatrix} - \begin{bmatrix} \{N^{\Delta T}\} \\ \{M^{\Delta T}\} \end{bmatrix}\right) \tag{9}$$

where $[A]$ and $[D]$ are the extensional and bending stiffness matrices, $\{N^{\Delta T}\}$ is the thermal force vector induced by the uniform temperature rise (T_u), and $\{M^{\Delta T}\}$ is thermal moment vector induced by the temperature gradient (T_g).

$$\{N^{\Delta T}\} = T_u\sum_{k=1}^{N}[\overline{Q}]_k\{\alpha\}_k(h_k - h_{k-1}) \tag{10a}$$