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A two-dimensional Fourier-series finite element for wrinkling analysis of thin films on compliant substrates

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ABSTRACT

In this paper, an efficient Fourier-series finite element solution framework is proposed to simulate the wrinkling phenomena in two-dimensional film/substrate system. In the method, the displacement field is transformed into the slowly variable Fourier coefficient, i.e., the macroscopic displacement field, which permits to capture the wrinkling evolution in the system with much less degrees of freedom than the full finite element model. The derived macroscopic non-linear system is solved by the Asymptotic Numerical Method that is very efficient and reliable to capture the bifurcation point and the post-buckling path in wrinkling analyses. In particular, the importance of using the first harmonic of Fourier series in approximating the axial displacement in substrate is discussed and a spurious phenomenon related to the hypothesis of the used approximation functions within the Fourier-series approach, i.e., oscillation locking, is pointed out. To overcome this phenomenon, modifications on either the Fourier series or the constitutive equations of the substrate are proposed. The efficiency and accuracy of the proposed macroscopic model are demonstrated by the wrinkling simulations for several kinds of film/substrate systems.

1. Introduction

Compressed stiff films bonded to a compliant substrate can buckle into a pattern presenting sinusoidal wrinkles with constant wavelength [1,2] when compression reaches a critical value. The wrinkles may have an undesirable effect on the system and should be often avoided. However, the periodic nature of the wrinkles in film/substrate systems has nowadays found some applications such as buckling-based metrology method [3], optical gratings [4] and stretchable electronics [5]. Since the critical compressive load and details of the instability pattern are often of interest, an accurate yet efficient characterisation of wrinkling of film/substrate system is necessary.

Wrinkling in film/substrates system is very similar to the symmetrical wrinkling in sandwich panel, in which wrinkling stresses for three modes (single sided face wrinkling, in-phase wrinkling and out-of-phase wrinkling) are expressed by a unified, single expression through approximate structural analysis [6,7]. Recently, some non-linear analyses have been developed to comprehend and characterise the wrinkle morphologies. The stiff thin films are usually modelled as non-linear elastic beams or plates in the works [8,2,1,9]. Differences in these works mainly arise from the used kinematics for the substrate and whether the shear deformation at the film/substrate interface is considered or not. Chen and Hutchinson [2] modelled the substrate as an elastic foundation of springs where the in-plane displacements are ignored. Results show that the herringbone mode constitutes a minimum energy configuration. Huang et al. [1] further adopted the three-dimensional elastic filed for the substrate and investigated the influence of the Young's modulus, the Poisson's ratio and the thickness of the substrate on the critical strain, amplitude and wavelength of the sinusoidal wrinkles. In their works, the shear deformation in the film/substrate interface was neglected. Mei et al. [10] assumed a continuous shear strain across the film/substrate interface. This showed that a significant error of the critical strain and the wrinkling wavelength emerges in Huang et al. [1] when the Poisson's ratio of the substrate decreases. In the above mentioned works, linear elastic constitutive law was assumed in the substrate. Song et al. [9] considered finite strains and a non-linear constitutive law in the substrate and showed that the wrinkling wavelength decreases with the increase of the pre-strain rather than remaining constant as in [1]. Furthermore, Brau et al. [11] pointed out that a quadratic non-linearity of the substrate can trigger

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the period-doubling instability that cannot be captured by a linear substrate. Hutchinson [12] also studied the influence of the quadratic non-linearity of a neo-Hookean substrate on the mode evolution of the wrinkles in film/substrate systems. Shariyat and Asemi [13] recently investigated the instability of rectangular FGM plate on elastic foundation under shear loading and found that the stiffness of the elastic foundation greatly affects the angles of deformation waves. Taking into account both accuracy and computational efficiency, higher order functions can be used to approximate kinematics of the substrate and thus to reduce the number of unknowns, see [14–18]. Yang et al. [19] proposed a higher-order kinematics to model substrate based on the Carrera's Unified Formulation (CUF) [20–22]. The finite element method was used to solve the higher-order model and accurate results were obtained with low computational cost.

In this paper, stemming from an efficient multi-scale approach established by Damil and Potier-Ferry [23,24] that exploits the periodic nature of the wrinkles, an effective two-dimensional Fourier-based model is developed to study the sinusoidal wrinkling in thin stiff films on compliant substrates. The problem unknowns arising from the assumed kinematics (here addressed as "microscopic model" where Euler-Bernoulli's beam theory is used for the film and a two-dimensional plane-strain solid for the substrate) are expanded by Fourier series, which leads to a new problem with the Fourier coefficients as new unknowns exhibiting much slower variation than the original unknowns. This latter problem is called "macroscopic model". The derived macroscopic model has the advantage to require very few degrees of freedom to accurately describe the problem under study. Compared to the Landau-Ginzburg technique, the Fourier-based method has two advantages: 1) not only the bifurcation point but also the post-buckling path can be captured and 2) the coupled global and local instability patterns can be incorporated and characterised, see Liu et al. [25]. The established nonlinear system shows strong nonlinearity near and after bifurcation point, and it is a difficult task to solve this kind of nonlinear problem. In this paper, the Asymptotic Numerical Method (ANM), known as an effective and robust nonlinear solver in tracing bifurcation path in instability problems proposed by Potier-Ferry et al. [26–28], is used to solve the established nonlinear equations. So far, the approach based on Fourier series has been successfully used for the wrinkling analysis of non-linear beams resting on Winkler's foundation [24,29], sandwich beams [30,31] and thin metal films [32–35]. For the above Fourier-related models, the neutral axial displacement is always assumed not fluctuating and the first order harmonics are ignored, which matches the practical kinematics in these beam or plate elements. However, in the current film/substrate model, the substrate is discretized by continuum elements and shows oscillation at the film/substrate interface. Therefore, the first harmonic of the axial displacement should be introduced to characterise such oscillation. Otherwise, a spurious stiffening effect (named as "oscillation locking") would occur if only the zero order of Fourier series for the axial displacement is used as introduced in previous works. To overcome this difficulty, two manners are proposed: (1) the first one is to enrich the Fourier-series expansion and (2) the second one is to modify the elastic coefficient of the substrate.

The paper is structured as follows. The two-dimensional microscopic model for film/substrate systems is introduced in Section 2. In Section 3, the Fourier-related macroscopic model and the corresponding finite element are derived. In Section 4, numerical tests are performed to assess the established macroscopic model, and discussion on the importance of the first harmonic in continuum element and the oscillation locking is made.

2. Microscopic model

2.1. Kinematics

A two-dimensional elastic stiff film bound to an elastic soft compliant substrate is considered as depicted in Fig. 1: x and z are the axial and through-the-thickness coordinates, h_f and h_s are the thickness of the top film and the substrate, respectively, h_t is the total thickness of the structure. The length and the width are denoted respectively by L and b. The thin film is modelled as an elastic Euler-Bernoulli's beam:

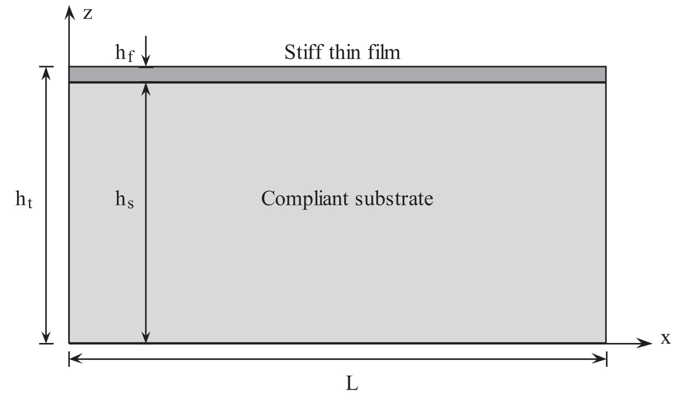

Fig. 1. Sketch of an elastic thin stiff film on a compliant substrate.

$$\begin{aligned} \mathcal{U}^f(x,z) &= u^f(x) - \left(z - \frac{h_f + 2h_s}{2}\right) w^f_{,x}(x) \\ \mathcal{W}^f(x,z) &= w^f(x) \end{aligned} \quad z \in [h_s, h_t] \tag{1}$$

where the superscript "f" stands for the film, $\mathcal{U}(x,z)$ and $\mathcal{W}(x,z)$ are the axial and through-the-thickness components of the displacement field $\boldsymbol{U}(x,z)$ and u^f and w^f the two unknown displacement functions. A coordinate subscript preceded by comm "$,x$" stands for a partial derivative. The substrate, denoted by "s", is modelled as a plane-strain elastic solid:

$$\begin{aligned} \mathcal{U}^s(x,z) &= u^s(x,z) \\ \mathcal{W}^s(x,z) &= w^s(x,z) \end{aligned} \quad z \in [0, h_s] \tag{2}$$

The displacement field at interface of the film and substrate should satisfy the following compatibility conditions:

$$\begin{aligned} \mathcal{U}^f(x,h_s) &= \mathcal{U}^s(x,h_s) \\ \mathcal{W}^f(x,h_s) &= \mathcal{W}^s(x,h_s) \end{aligned} \tag{3}$$

The Constrained Variational Principle (CVP) is used to ensure the congruency of the displacement field at the interface:

$$\Gamma = \{(x,y,z): x \in [0,L],\ \ y \in [-b/2, b/2],\ \ z = h_s\} \tag{4}$$

The following constrain term $\mathcal{L}(\boldsymbol{\mu}, \mathbf{u})$ is obtained by introducing the Lagrange multipliers $\boldsymbol{\mu} = \{\mu_1, \mu_2\}$ as fictitious gluing forces:

$$\begin{aligned} \mathcal{L}(\boldsymbol{\mu}, \mathbf{u}) = \int_\Gamma \{\mu_1[\mathcal{U}^f(x,h_s) - \mathcal{U}^s(x,h_s)] + \mu_2[\mathcal{W}^f(x,h_s) \\ - \mathcal{W}^s(x,h_s)]\} d\Gamma \end{aligned} \tag{5}$$

2.2. Geometric equations and constitutive law

The geometric equations and constitute law are supposed to meet the following hypotheses:

1. the material behaviour is linear elastic and described by the Hooke's law,
2. the geometrical non-linearity is considered in the film only and the strain-displacement relationship is described by the Green-Lagrange strain,
3. the substrate undergoes small displacements.

The above assumptions are translated into the following equations: