Thin-Walled Structures 115 (2017) 76–85

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Load-carrying capacity of thin-walled composite beams subjected to pure bending

Adrian Gliszczyński*, Tomasz Kubiak

Department of Strength of Materials Lodz University of Technology Mechanical Engineering Faculty, Stefanowskiego 1/15, Lodz 90-924, Poland

ARTICLE INFO

Keywords:
Failure criteria
Load-carrying capacity
FEM
GFRP
Thin-walled structures
Pure bending

ABSTRACT

The paper deals with estimating load capacity of thin-walled composite beams with C-shaped cross-section subjected to pure bending. The discussed beams were made of eight-layer GFRP laminate. The analysis have been performed on six ply systems. To designate load capacity of analyzed structures the ANSYS® program based on finite element method were employed. The experimental tests were conducted. Estimations of load capacity were based on the following failure criteria: Inverse of Tsai-Wu, Hoffman and criterion of maximum stress reduced to fiber direction. Based on the performed experimental and numerical studies it has been concluded that the largest convergence of numerical and experimental results was obtained with the implementation of the criterion of maximum stress reduced to the fiber direction. After exceeding the compressive strength or tension strength in the direction of the fibers the composite beams were characterized by a high stiffness degradation, leading to rapid destruction of the structures.

1. Introduction

The thin-walled structures are made of steel [1–5], composites [6,7], reinforced concrete [8,9], and nowadays more frequently as hybrids [10], or even as steel elements reinforced locally by composite materials [11]. Among the wide area of potential applications of thin-walled structures mainly sports, automotive and aerospace industries should be indicated. In the world literature there are many papers on the stability, postbuckling behavior and load carrying capacity of composite structures in which these issues were solved using analytical, numerical and analytical-numerical methods [12–15]. The results of numerical calculations published in the papers of various authors were confronted with the results of experimental studies [16–18]. Laminates are currently the most common type of composites and for many years have been applied in girder structures of aircraft wings [19], helicopter blades [20] or girders of wind turbine blades [21].

The presented work focuses on estimating the load carrying capacity of thin-walled composite beams subjected to pure bending. Particular attention is paid to determine the loads and the forms of failure for all considered beams. In the authors' opinion, there are not enough papers devoted to the results of experimental studies on load-carrying capacity and the failure range of composite profiles subjected to operating loads in the world literature. Therefore, it has been decided to analyzed and describe the failure of thin-walled laminate structures. The thin-walled channel section profile made of an epoxy resin laminate reinforced with glass fibers (GFRP) has been taken into consideration. The results presented in this paper concern the six arrangements of layers. Both the failure values of load and form of failure determined from numerical studies have been compared with experimental results.

2. The analyzed failure criteria

In opposition to isotropic materials, for which a well known strength hypothesis e.g. Huber–Mises–Hencky hypothesis or maximum shear stress hypothesis can be employed, defining the load carrying capacity of orthotropic structure is a much more complex task. This is due to fact that principal directions of stress and strain tensors are not coaxial. Moreover, in the case of isotropic materials analysis, to determine the safe working range of structure, knowledge of the certain values of maximal stress, which are appointed in uniaxial tensile tests is sufficient. For orthotropic material in a plane stress state it is necessary to know until five material constants: tensile strength along the fibers direction T_1, tensile strength in the transverse direction to fibers T_2, compressive strength along the fibers direction C_1, compressive strength in the transverse direction to fibers C_2 and the shear strength in the plane of the principal axes of orthotropy S_{12}. The precise number of failure criteria formulated for composite materials is virtually impossible to determine because still appear in the world literature new proposals. In the paper written by A. Muc [22] it can be found that the number of different failure criteria for composite materials may

* Corresponding author.
E-mail addresses: adrian.gliszczynski@dokt.p.lodz.pl (A. Gliszczyński), tomasz.kubiak@p.lodz.pl (T. Kubiak).

http://dx.doi.org/10.1016/j.tws.2017.02.009
Received 10 October 2016; Accepted 10 February 2017

even be several hundred.

2.1. The criterion of maximum stress

Maximum stress criterion assumes that the condition for safe state of structure is that the stresses in the principal directions of the material orthotropy (1–2) are lower than the corresponding strength (index “1” denotes the fibers direction and index “2” – direction transverse to the fibers direction):

$$-C_1 < \sigma_1 < T_1, \qquad -C_2 < \sigma_2 < T_2, \qquad -S_{12} < \sigma_6 < S_{12} \tag{1}$$

where:

C_1, T_1 - compressive and tensile strength in direction 1, respectively.

C_2, T_2 - compressive and tensile strength in direction 2, respectively (orthogonal to direction 1).

S_{12} - shear strength in the plane of orthotropy (1–2).

If all six inequalities (1) [23] are satisfied then, according to maximal stress criterion the analyzed material is not destroyed. However, even if one of these inequalities is not satisfied then the material is destroyed. A distinct disadvantage of this criterion is fact that this description does not take into consideration the coupling between the normal and shear stresses. Nevertheless, in engineering practice criterion of maximum stress is one of the most popularized failure criteria, as evidenced by its application in many standards for composite structures [24].

2.2. The criterion of maximum stress reduced to fiber direction (MSRFD)

The criterion of maximum stress reduced to fiber direction (MSRFD) determines the exhaustion of load carrying capacity by exceeding the acceptable stresses solely in the direction of the fibers. The acceptance of this principle is equivalent with a statement that despite of the devastation in the matrix, a beam is not subject to destruction, because still the loads applied to the structure are carried by the fiber. The described condition is therefore in fact a modification of the criterion of maximum stress, which reduces the area of strength analysis only to study the behavior of layers in the direction of the fibers. Destruction condition can be formally written in the form of the following inequality:

$$-C_1 < \sigma_1 < T_1 \tag{2}$$

For the purpose of the observation maps of the strength factor f distributions, inequalities (2) have been defined as in Hashin's criterion in the following form (3) i (4):

$$\sqrt{\left(\frac{\sigma_1^2}{T_1^2}\right)} \geq 1 \quad - \quad \text{fiberdestruction(fortensilestresses)}, \tag{3}$$

$$\sqrt{\left(\frac{\sigma_1^2}{C_1^2}\right)} \geq 1 \quad - \quad \text{fiberdestruction(forcompressivestresses)}. \tag{4}$$

2.3. Interactive criteria

In 1971, Tsai and Wu proposed the interaction strength criterion, which in their description connected the correlation between the normal and shear stresses in complex stress. In the Voight's notation, the criterion of Tsai-Wu describing the destruction in a three-dimensional state of stress has the following form (5) [25]:

$$F_i\sigma_i + F_{ij}\sigma_i\sigma_j = 1 \quad i, j = 1, 2, \ldots, 6 \tag{5}$$

Due the fact that each considered wall of composite beams was in plane stress, Eq. (5) can be written in the following form (6):

$$F_1\sigma_1 + F_2\sigma_2 + F_{11}\sigma_1^2 + 2F_{12}\sigma_1\sigma_2 + F_{22}\sigma_2^2 + F_{66}\sigma_6^2 = 1 \tag{6}$$

where:

$$F_1 = \frac{1}{T_1} - \frac{1}{C_1} \qquad F_2 = \frac{1}{T_2} - \frac{1}{C_2} \qquad F_{11} = \frac{1}{T_1C_1}$$
$$F_{22} = \frac{1}{T_2C_2} \qquad F_{12} = -\frac{1}{2}\sqrt{\frac{1}{T_1C_1T_2C_2}} \qquad F_{66} = \frac{1}{S_{12}^2} \tag{7}$$

The meaning of parameters T_1, T_2, C_1, C_2, S, is analogous to the quantities described for the criterion of maximum stress. As it can be seen in the case of plane stress, virtually all parameters at normal and shear stresses in the formula (6) can be determined in uniaxial compression, tension or shear tests. The exception in this case is the parameter F_{12}, whose interpretation is the interaction between the normal stresses σ_1 and σ_2. For the set values of the stresses σ_1 and σ_2, this coefficient can be determined in a biaxial test. Universal determination of this parameter is currently still an unresolved issue. The value of parameter F_{12} is characterized by variability in different ratio of stress σ_1 to σ_2, it also changes as a result of the character acting pairs of stresses (biaxial tension, biaxial compression, tension with compression). Out of the six Eq. (7) a characteristic element of the Tsai-Wu criterion is the declaration of the parameter F_{12}. Despite the identity of the mathematical descriptions for most of the interactive criteria, individuality in the definition of the coefficient F_{12} constitutes the starting point for many definitions of failure criteria for orthotropic materials well-known in strength of materials. For further numerical analyses has been the Hoffman criterion selected [26], where:

$$F_1 = \frac{1}{T_1} - \frac{1}{C_1} \qquad F_2 = \frac{1}{T_2} - \frac{1}{C_2} \qquad F_{11} = \frac{1}{T_1C_1}$$
$$F_{22} = \frac{1}{T_2C_2} \qquad F_{12} = \frac{1}{T_1C_1} \qquad F_{66} = \frac{1}{S_{12}^2} \tag{8}$$

Implemented in the numerical program, the Inverse of Tsai-Wu criterion described as the strength factor f, is written in the following form [37] (9):

$$f = \left(-\frac{B}{2A} + \sqrt{\left(\frac{B}{2A}\right)^2 + \frac{1}{A}}\right)^{-1} \tag{9}$$

where:

$$A = \frac{\sigma_1^2}{T_1C_1} + \frac{\sigma_2^2}{T_2C_2} + \frac{\sigma_{12}^2}{S_{12}} + \frac{\sigma_1\sigma_2}{\sqrt{T_1C_1T_2C_2}} \tag{10}$$

$$B = \left(\frac{1}{T_1} - \frac{1}{C_1}\right)\sigma_1 + \left(\frac{1}{T_2} - \frac{1}{C_2}\right)\sigma_2 \tag{11}$$

The Tsai-Wu coupling coefficients were assumed as "−1" and it should be noted that the compression strength in (2) and (3) are here positive numbers (cf. Table 2). Numerical values of load carrying capacity of individual layers were determined by specifying two successive load steps M_i and M_{i+1} between which the conditions of individual criteria are exceeded. This situation is accompanied by a change of strength factor values from the value $f_i < 1$ to the value $f_{i+1} > 1$. Applying linear interpolation between successive load steps M_f point was determined that in light of the assumptions defined the destruction of the first layer (lower estimate), or the destruction of all the layers in the laminate (upper estimate). The value of the failure load (bending moment) can be described as the following formula:

$$M_f = \frac{M_i(1 - f_{i+1}) - M_{i+1}(1 - f_i)}{f_i - f_{i+1}} \tag{12}$$

where:

Table 1
Assumed autoclaving parameters for the manufacturing of composite laminate structures.

Curing temp. [°C]	Heating/cooling rate [°C/min]	Curing time [min]	Pressure [MPa]	Vacuum [MPa]
100	1	60	0.4	0.085