Thin–Walled Structures 115 (2017) 240–246

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Full length article

The effect of heterogeneity on the parametric instability of axially excited orthotropic conical shells

A.H. Sofiyev[a,*], E.B. Pancar[b]

[a] Department of Civil Engineering of Engineering Faculty of Suleyman Demirel University, Isparta, Turkey
[b] Department of Civil Engineering of Engineering Faculty of Ondokuz Mayis University, Samsun, Turkey

ARTICLE INFO

Keywords:
Parametric instability
Time dependent axial compressive load
Heterogeneity
Orthotropic material
Truncated conical shell
Backward and forward excitation frequencies

ABSTRACT

The parametric instability of heterogeneous orthotropic truncated conical shells under time dependent axial compressive load on the basis of Donnell shell theory is investigated. The dynamic basic equations are reduced into a Mathieu-Hill type differential equation describing the instability of heterogeneous orthotropic truncated conical shells using Galerkin's method. The backward and forward excitation frequencies are determined by using Bolotin's method. A comparison with the previous studies has been developed in order to validate the present results. The effects of axial load factors, heterogeneity, orthotropy, as well as the variation of the characteristics of the conical shell on the backward and forward excitation frequencies are studied in detail.

1. Introduction

Among the shell structures, the conical shells stand out sharply from the other structural forms and are widely used in the aerospace, marine and other industries. As a result, much research has been devoted to the study of the static and dynamic characteristics of conical shells at various loading conditions. The numerical analyses of the mechanical response of conical shells under periodic loads are a recurrent topic in these specific studies. The parametric resonance of conical shells caused by periodic loading can occur when the values of load is much smaller than the static buckling load. So the shell components are designed to withstand a static buckling fail in a periodic loading. In addition, the instability of shell structures occurs in the forcing frequency range, leading rather than at a single value. The parametric response of conical shells under pulsating pressure has been investigated since the first paper published by Kornecki [1]. Following this study, the parametric vibration or instability problems of homogeneous isotropic conical shells with periodic loads have been carefully investigated [2–8]. Sahu and Datta [9] presented an extensive bibliography of works on the dynamic instability of plates and shells from 1987 to 2005.

The number of works belonging to parametric responses of anisotropic shells is relatively scarce and the majority of these studies related to composite cylindrical shells. One of the first study on the dynamic instability of anisotropic shells is given by Goroshko and Emelyanenko [10]. Bert and Birman [11] studied the parametric instability of thick orthotropic shells using higher order shell theories. Argento [12] studied the dynamic stability of composite circular clamped shells under axial and torsional loading using the Donnell's linear theory. Argento and Scott [13,14] determined the instability regions of a composite (graphite/epoxy) circular cylindrical shell subjected to periodic loads using a perturbation technique. Ng and Lam [15] presented the dynamic instability of laminated composite cylindrical shells subjected to periodic axial loads. Jansen [16] developed an analytical simplified approach in order to simulate dynamic step and periodic axial loads acting on the anisotropic shells. Mallon et al. [17] studied orthotropic circular cylindrical shells using Donnell's shell theory and they also presented experimental results. Ovesy and Fazilati [18] investigated the parametric instability regions of laminated composite plate and cylindrical shells subjected to non-uniform in-plane axial end-loadings. Dey and Ramachandra [19] analyzed static and dynamic instability of composite cylindrical shell panels subjected to partial edge loading. Panda et al. [20] presented hygrothermal response on the parametric instability of delaminated bidirectional composite flat panels.

Recently, material scientists have shown that the material efficiency can be significantly improved if their composition and structure are varied to meet their functional requirements. Such heterogeneous materials have engineered gradients of composition or structure, which offer superior performance over conventional homogeneous materials. The shells made of heterogeneous materials are widely used in space vehicles, aircrafts, nuclear power plants and many other engineering applications. Therefore, some studies have been conducted to investigate the parametric vibrations of heterogeneous shells. A review of the literature shows that the great majority of the investigations have been

* Corresponding author.
E-mail address: abdullahavey@sdu.edu.tr (A.H. Sofiyev).

http://dx.doi.org/10.1016/j.tws.2017.02.023
Received 4 January 2017; Received in revised form 20 February 2017; Accepted 23 February 2017
0263-8231/

limited to the parametric vibration or dynamic instability behaviors of heterogeneous isotropic or functionally graded (FG) isotropic shells. The first study on the dynamic instability of heterogeneous isotropic truncated conical shells with variable modulus of elasticity, under compressive axial forces which are periodic in the time, using Donnell shell theory examined by Massalas et al. [21]. In recent years, the development of technology has greatly simplified the production of functional graded materials (FGMs), and this contributed to the creation of noteworthy work on the instability of conical shells consisting of FG isotropic materials [22–33]. It is noted that the number of studies on the free vibration problems of heterogeneous orthotropic and isotropic shells is sufficiently greater than the parametric vibrations in the literature [34–48].

A literature search revealed that the study of parametric vibration of heterogeneous orthotropic truncated conical shells under the time-dependent periodic axial load is absent. The purpose of this study is to examine this issue in detail.

2. Basic relations

Assume that the heterogeneous orthotropic truncated conical shell with the thickness h, the slant length L, the semi-vertex angle γ, the small and large mean radii R_1 and R_2, and the distances from the vertex to small and large bases S_1 and $S_2 = S_1 + L$, respectively, is subjected to uniformly distributed edge forces, as illustrated in Fig. 1, with the resultant axial load:

$$N_S^0 = -N(t) = -N_0 - N_t\cos(\Omega t), \qquad N_\theta^0 = 0, \quad N_{S\theta}^0 = 0 \tag{1}$$

where N_S^0, N_θ^0 and $N_{S\theta}^0$ are the membrane forces for the condition with zero initial moments, N_0 is the uniform static axial load, N_t is the amplitude of the time dependent periodic axial load, while the frequency Ω is the frequency of excitation in radians per time unit and t is a time variable.

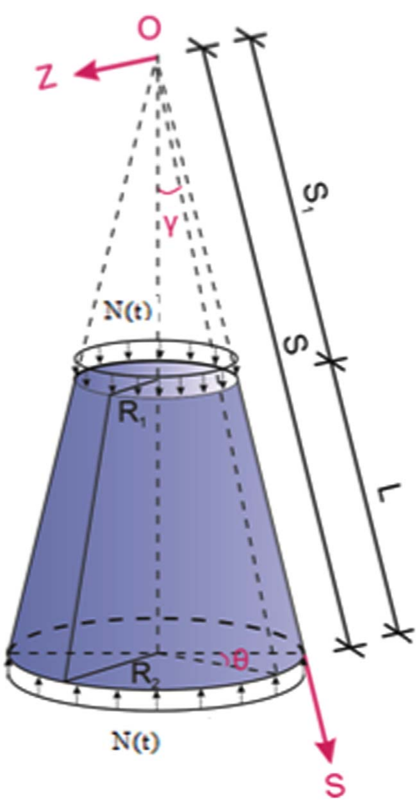

Fig. 1. Heterogeneous orthotropic truncated conical shell under time dependent periodic axial load.

The curvilinear coordinate system is defined as $(OS\theta z)$, where S and θ coincide with generator and circumferential directions, respectively, and z is normal to the $S\theta$ surface and its direction is inwards normal of the heterogeneous orthotropic truncated conical shell.

The material properties of heterogeneous truncated conical shells are assumed to have in-plane orthotropy and transverse heterogeneity. The orthotropic material properties vary c direction of the truncated conical shell and mathematically formulated as [34,39].

$$E_1(Z) = \phi(Z)E_{01}, \quad E_2(Z) = \phi(Z)E_{02}, \quad G(Z) = \phi(Z)G_0, \quad Z = z/h \tag{2}$$

where E_{01} and E_{02} are the Young's moduli in the S and θ directions, respectively, G_0 is the shear modulus and Z is the normalized thickness coordinate, $-1/2 \le Z \le 1/2$. $\phi(Z)$ is continuous heterogeneity function and explanation about it is given in the discussion part in detail. The density, ρ_0, and Poisson's ratios, ν_{12} and ν_{21}, through the thickness coordinate are assumed to be constant. In addition, Young's moduli and Poisson's ratios are related by the following expression, $\nu_{21}E_{01} = \nu_{12}E_{02}$ [49].

According to the classical shell theory (CST), the stress-strain relations for heterogeneous orthotropic truncated conical shells are given as follows:

$$\begin{aligned}
\sigma_S &= K_{11}e_S + K_{12}e_\theta - z\left[K_{11}\frac{\partial^2 w}{\partial S^2} + K_{12}\left(\frac{1}{S^2}\frac{\partial^2 w}{\partial \theta_1^2} + \frac{1}{S}\frac{\partial w}{\partial S}\right)\right]\\
\sigma_\theta &= K_{21}e_S + K_{22}e_\theta - z\left[K_{21}\frac{\partial^2 w}{\partial S^2} + K_{22}\left(\frac{1}{S^2}\frac{\partial^2 w}{\partial \theta_1^2} + \frac{1}{S}\frac{\partial w}{\partial S}\right)\right]\\
\sigma_{S\theta} &= K_{66}e_{S\theta} - zK_{66}\left(\frac{1}{S}\frac{\partial^2 w}{\partial S\partial\theta_1} - \frac{1}{S^2}\frac{\partial w}{\partial\theta_1}\right)
\end{aligned} \tag{3}$$

where σ_S, σ_θ, $\sigma_{S\theta}$ are the stresses, e_S, e_θ, $e_{S\theta}$ are the strains, $\theta_1 = \theta\sin\gamma$, w is the displacement of the middle surface in the normal direction, positive towards the axis of the cone and assumed to be much smaller than the thickness and the quantities for the heterogeneous orthotropic materials, K_{ij}, $(i, j) = (1, 2, 6)$, are defined as [39]:

$$K_{11} = \frac{E_{01}\phi(Z)}{1 - \nu_{S\theta}\nu_{\theta S}}, \quad K_{22} = \frac{E_{02}\phi(Z)}{1 - \nu_{S\theta}\nu_{\theta S}}, \quad K_{12} = \nu_{\theta S}K_{11} = \nu_{S\theta}K_{22} = K_{21},$$
$$K_{66} = 2G_0\phi(Z) \tag{4}$$

The force and moment resultants of shells are expressed by the following relations [49]:

$$[(N_S, N_\theta, N_{S\theta}), (M_S, M_\theta, M_{S\theta})] = \int_{-h/2}^{h/2}(\sigma_S, \sigma_\theta, \sigma_{S\theta})[1, z]\mathrm{d}z \tag{5}$$

The Airy stress function, $\Psi(S, \theta, t)$, is introduced by the following relations [50]:

$$(N_S, N_\theta, N_{S\theta}) = \left(\frac{1}{S^2}\frac{\partial^2\Psi}{\partial\theta_1^2} + \frac{1}{S}\frac{\partial\Psi}{\partial S}, \quad \frac{\partial^2\Psi}{\partial S^2}, \quad -\frac{1}{S}\frac{\partial^2\Psi}{\partial S\partial\theta_1} + \frac{1}{S^2}\frac{\partial\Psi}{\partial\theta_1}\right) \tag{6}$$

3. Basic equations

The governing differential equations for dynamic stability of truncated conical shells modified for the heterogeneous truncated conical shells can be written as [39,50]:

$$\begin{aligned}
&\frac{\partial^2 M_S}{\partial S^2} + \frac{2}{S}\frac{\partial M_S}{\partial S} + \frac{2}{S}\frac{\partial^2 M_{S\theta}}{\partial S\partial\theta_1} - \frac{1}{S}\frac{\partial M_\theta}{\partial S} + \frac{2}{S^2}\frac{\partial M_{S\theta}}{\partial\theta_1} + \frac{1}{S^2}\frac{\partial^2 M_\theta}{\partial\theta_1^2} + \frac{N_\theta}{S}\cot\gamma\\
&+N_S^0\frac{\partial^2 w}{\partial S^2} + \frac{N_\theta^0}{S}\left(\frac{1}{S}\frac{\partial^2 w}{\partial\theta_1^2} + \frac{\partial w}{\partial S}\right) + 2N_{S\theta}^0\frac{\partial}{\partial S}\left(\frac{1}{S}\frac{\partial w}{\partial\theta_1}\right) - \rho_0 h\frac{\partial^2 w}{\partial t^2} = 0
\end{aligned} \tag{7}$$

$$\frac{\cot\gamma}{S}\frac{\partial^2 w}{\partial S^2} - \frac{2}{S}\frac{\partial^2 e_{S\theta}}{\partial S\partial\theta_1} - \frac{2}{S^2}\frac{\partial e_{S\theta}}{\partial\theta_1} + \frac{\partial^2 e_\theta}{\partial S^2} + \frac{1}{S^2}\frac{\partial^2 e_S}{\partial\theta_1^2} + \frac{2}{S}\frac{\partial e_\theta}{\partial S} - \frac{1}{S}\frac{\partial e_S}{\partial S} = 0 \tag{8}$$