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Comparative study between XFEM and Hashin damage criterion applied to failure of composites

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A B S T R A C T

This paper presents a comparative study between Hashin damage criterion and the eXtended Finite Element Method (XFEM) applied to the failure of fiber reinforced polymers (FRP). A brief literature review on failure criteria to predict the failure of FRP is firstly presented. Then, finite element models of square plates with different layer configurations, containing a circular hole with distinct radii and subjected to monotonic uniaxial tension are described within the framework of ABAQUS package. The models are validated by comparison between the numerical results and those of a benchmark model. Finally, the influence of (i) stacking sequence, (ii) hole radii and (iii) failure criteria (Hashin and XFEM) on the load *vs.* elongation paths, stresses distributions and collapse configurations of the plates is shown and discussed and some conclusions are drawn.

1. Introduction

The increasing demand for reinforced polymers (FRP) is due to their low self-weight, high strength, reduced maintenance and fast manufacture/construction, which constitute great advantages in many structural applications [1]. On the other hand, FRP often exhibit high sensitivity to cracking and interlaminar delaminations and are generally characterized by brittle failures. Because ductile behavior is usually absent in FRP structures and their collapse occurs suddenly, the prediction of failure of FCR structural members is of utmost importance. Additionally, their inherent anisotropy also contributes to the complexity of constitutive stress-strain laws [2]. Widely accepted as the most powerful numerical technique to analyze the behavior of structures, the Finite Element Method (FEM) has been intensively used to study and predict the behavior of FRP structures as well as their collapse through the implementation of different failure criteria.

The first generation of failure criteria for FRP was based on those criteria developed for isotropic materials, such as metals (e.g. von Mises criterion). Such simple criteria (e.g., Tsai-Hill and Tsai-Wu [3,4]) only extended the scope of von Mises criterion to composite materials, thus including the orthotropic nature of these materials. However, it is known that both criteria (Tsai-Hill and Tsai-Wu) are capable to identify the failure of a material point but are unable to influence that failure on the degradation of material stiffness. In other words, these first generation criteria could only detect the first failure of a given structural element. Beyond this first failure equilibrium state, the analysis remains fully elastic without drop of strength and loses accuracy. These failure criteria are also not associated with any type of failure mode and the single failure index is usually written in a polynomial form.

The second generation of failure criteria also mimic the principals of its preceding generation. However, they influence the stiffness and strength of the FRP by degrading these properties beyond material failure. Among several criteria, the Hashin criterion [5,6] deserves to be credited. Hashin criterion is associated with different types of failure modes (fiber, matrix, shear), each mode associated with a distinct failure index, each index defined by a function of ultimate stresses. Unlike the first generation of failure criteria, Hashin-based criterion includes a progressive damage model in which the stiffness (constitutive matrix) of a damaged point is decreased through the consideration of the previously calculated failure mode indexes. Besides, the progressive damage model also takes into account the brittle nature of FCR by considering its fracture energy through the adoption of stress-displacement curves. After the onset of failure, the damage variables tend to unit while the equivalent stress components tend to zero. Therefore, the load-displacement curves associated to the brittle response of FRP may present severe drops of applied load for nearly zero increase of displacements. This behavior is consistent to what occurs in practice but it also might fail to simulate correctly the collapse mode of the structural member, i.e. the physical (brittle) separation of material, the appearance of cracks and their evolution with the applied load is totally absent in Hashin-based criterion. Recently, another failure criterion considering distinct types of failure modes, the Discrete Damage Model (DDM), has been proposed by Barbero and Cortes [7]

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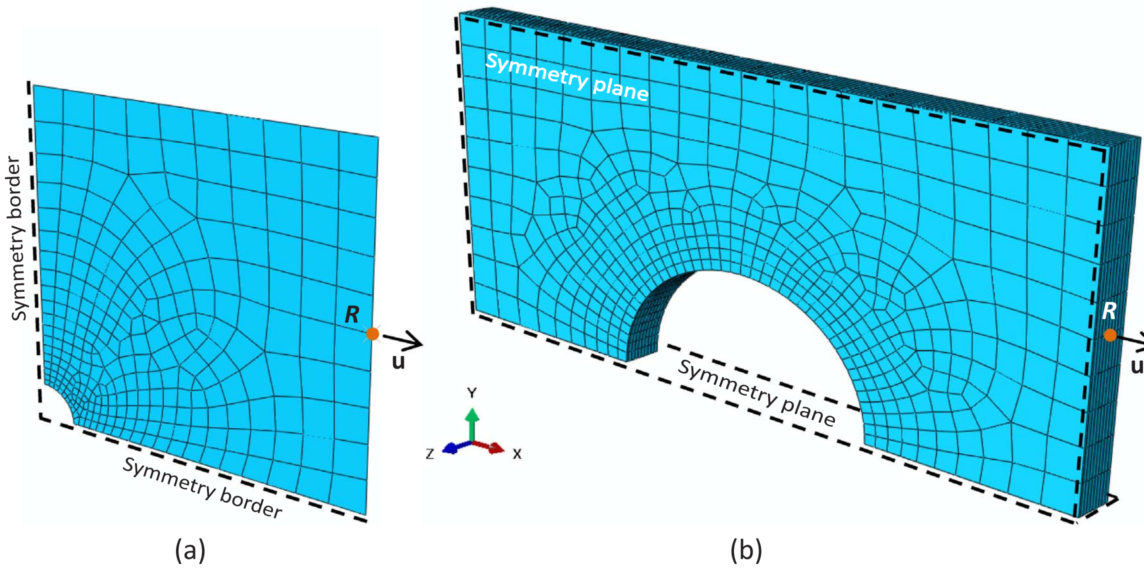

Fig. 1. FE geometry and mesh of (a) Hashin-based models (r = 1.25 mm) and (b) XFEM-based models (r = 5.00 mm).

and then modified by Moure et al. [8] to include fiber breakage and, thus, the failure of laminates. In DDM, the fiber strength is determined by a Weibull stochastic distribution and matrix cracking is seeded by assigning initial crack densities in the plies. This model has been successfully validated against experimental data and extensively used by Moure et al. [8,9] to study aspects such as stiffness reduction of plates due to damage, stress intensity factors and the influence of cluster ply sequences in laminates with holes subjected to uniaxial tension. In DDM, cracks are represented by the evolution of the crack density parameter and the model is only applicable to plates with arbitrary, yet symmetric, ply distributions.

The third generation of failure criteria is now capable of considering the emergence of cracks in failed material points and also study their propagation in FRP with the applied load. Probably, the most well-known method is the Extended Finite Element Method (XFEM) [10,11]. In XFEM, a damage evolution parameter is defined for a given damage initiation criterion. Once damage is initiated according to that criterion, the damage parameter is set null and a crack emerges in the FE. Then, by considering a fracture energy based damage evolution law, as the strain in the FE is increased, the damage variable tends to *one* and the stress in the FE tends to *zero*. Crack opening and propagation is modeled using the phantom node method. Prior to damage initiation in a FE, phantom nodes with additional degrees of freedom are overlaying their corresponding real nodes. After damage is initiated, the FE is split into element fragments and the phantom nodes start to displace from the real nodes as the crack surface opens.

Despite being intensively used to study the strength of structures, Hashin damage criterion [2,12–14] and XFEM [15–18] have rarely been compared. Moreover, in the scope of FRP structures, these techniques were always compared with analytical and/or experimental results but not with each other. To the authors' knowledge, only the work done by Feerick et al. [19], dealing with prediction of crack initiation and propagation in cortical bone, effectively compared both XFEM and Hashin damage criterion. Since both methods are extensively used in the prediction of strength and failure of FRP structures, the originality and main objective of this paper is to compare both methods to analyze FRP structures and, taking into account the obtained results, to assess their differences, advantages and limitations. First, the numerical models of 24 square (25×25 mm^2) plates with a circular hole in the centre, comprising four different stacking sequences, three distinct hole radii and analyzed in the scope of the Hashin damage criterion and the XFEM, are described in detail. Then, the models are validated by comparison of numerical results with those of a benchmark model. The numerical results are then analyzed in detail with emphasis on the influence of the (i) stacking sequences, (ii) hole radii and (iii) failure criteria on the elastic stiffness and maximum load (load *vs.* elongation paths), stress distribution and collapse configuration of the plates. Finally, some conclusions are drawn.

2. Description of numerical models

In this section, the finite element models are described in the scope of ABAQUS [20] commercial package. The models consist of square (25×25 mm^2) composite plates with a circular hole in their centre, subjected to monotonic uniaxial tensile loading, with four different stacking sequences and three different hole radii. The plates' geometries are similar to those studied by Moure et al. [8]. They are composed by 19 stacked plies of a glass/vinylester laminate (Fiberite/HyE 9082Af) [21], each ply possessing a thickness t = 0.144 mm. The four studied stacking sequences have the following labeling: $[0/90_8/0_{1/2}]_S$, $[90_8/0,0_{1/2}]_S$, $[90/0_8/90_{1/2}]_S$ and $[0_8/90/90_{1/2}]_S$ where "0_N" and "90_N" are the number (N) of plies having fibers in 0° and 90° directions, respectively (i.e. parallel and perpendicular to the loading direction). The former and the latter will also be addressed, from this point on and for simplicity sake, as $plies_O$ and $plies_{9O}$, respectively. Additionally, in the presented labeling, the subscript "S" means that the plates are symmetric with respect to plate mid-plane. The three studied hole radii (r) are r = 1.25, r = 2.50 and r = 5.00 mm.

In order for the numerical solutions to be obtained, static monotonic non-linear analyses of the models were performed using an incremental-iterative scheme. For each stacking sequence and hole radius, two modeling approaches were developed and implemented: models analyzed in the scope of the Hashin damage initiation criteria [6] (Hashin-based models) and models analyzed in the scope of XFEM (XFEM-based models) [20]. Hence, a total of 24 finite element (FE) models were developed and analyzed, as described in the following sections.

2.1. Geometry, FE mesh and boundary conditions

Due to the different nature of the two modeling approaches (Hashin-based and XFEM-based models) a set of distinct modeling options were adopted regarding the geometry and FE mesh of the models, described next and presented in Fig. 1. In Fig. 1(a), the model geometry and FE mesh used in the scope of the Hashin damage initiation criterion are presented. In Fig. 1(b), the model geometry and FE mesh used in the scope of the XFEM are shown. Due to the symmetry of the studied