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A constrained spline finite strip method for the mode decomposition of cold-formed steel sections using GBT principles

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ABSTRACT

In this paper, an analysis procedure has been presented for cold-formed steel sections, for decomposing the buckling modes obtained using spline finite strip method (SFSM) into their primary and independent buckling modes such as local, distortional and global buckling. This procedure utilizes principles of generalized beam theory (GBT) to evaluate the restraint matrices corresponding to different modes. The restraint matrices are integrated in to spline stiffness matrices using transformation technique to extract pure modes corresponding to local, distortional and global modes. The proposed analysis technique termed as constrained spline finite strip method (cSFSM) has been validated for cold-formed steel open cross-sections subjected to axial compression and bending under various boundary conditions. The results are in agreement with buckling stresses evaluated using constrained finite strip method (cFSM) and GBT.

1. Introduction

Evaluation of elastic buckling stresses corresponding to global, distortional and local buckling is a prerequisite for the design of cold-formed steel structural members using Direct Strength Method (DSM). Elastic buckling stresses can be calculated using analytical expressions or by using numerical methods like finite element method (FEM), finite strip method (FSM) or by using generalized beam theory (GBT). The finite strip method (FSM) commonly used for the analysis of thin walled steel structures is a combination of Ritz-Galerkin approach and finite element concept which choses trigonometric function in longitudinal direction and polynomial interpolation function in transverse direction. The main disadvantage of this method is the infinite continuity of the longitudinal interpolation function, which makes it difficult in implementing complex boundary conditions and discontinuities. The continuity and discontinuity requirements during the analysis can be satisfied using spline finite strip method (SFSM) by replacing the trigonometric function in longitudinal direction with spline function.

The spline finite strip method (SFSM) was introduced for the analysis of elastic thin plates and shallow shells with various interior and boundary conditions [1]. SFSM has been used for buckling analysis by Lau and Hancock [2] on thin plates and thin-walled structures subjected to longitudinal compression and bending, transverse compression and shear by incorporating various boundary conditions. This method was extended for inelastic buckling analysis of thin-walled structural members and plates by taking the non-linear material stress-strain properties, strain hardening and residual stresses into account [3]. Later Hancock et al. [4] performed buckling and nonlinear analysis of thin-walled members undergoing local, distortional and overall buckling incorporating full nonlinear response with post-local buckling and plasticity using spline finite strip and semi-analytical finite strip method. A nonlinear elastic analysis based on spline finite strip method has been developed by Kwon and Hancock [5] for handling local, distortional and overall buckling modes in the post-buckling range and allowing geometric imperfections, arbitrary loading and non-simple boundary conditions. Elasto-plastic large deflection analysis of cold-formed steel members was performed using nonlinear spline finite strip method based on total Lagrangian approach by incorporating geometric and material nonlinearity, initial imperfections and residual stresses [6]. Iso-parametric spline finite strip formulation has been presented by Eccher et al. [7,8] for elastic buckling analysis and geometric nonlinear analysis of perforated folded plate structures. Yao and Rasmussen [9,10] presented iso-parametric spline finite strip method for material inelastic and geometric nonlinear analysis of perforated thin-walled steel structures by including nonlinear solution techniques, inelastic material models, selective reduced integration strategies, convergence criteria and solution procedures. Pham and Hancock [11] performed buckling analysis of lipped channel section under shear using semi analytical finite strip method and spline finite strip method.

The buckling analysis using SFSM as well as other methods like FSM and FEMyield several buckling modes, out of these, the buckling mode corresponding to local, distortional and global buckling has to be

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identified for design using DSM. The generalized beam theory (GBT) can be used to identify the buckling modes automatically, since the method is generalization of classical beam theory in which various deformation modes are used as basis functions. To decompose the buckling modes in FSM, a procedure known as constrained finite strip method (cFSM) based on GBT basic assumptions has been proposed for global and distortional buckling modes by Ádány and Schafer [12,13]. The procedure was extended by Ádány and Schafer [14] by decomposing local, shear/transverse extension modes by including sub nodes in addition to global and distortional buckling modes. The cFSM procedure has been applied to closed and branched cross-sections by Djafour et al. [15] for decomposing local buckling mode from combined global-distortional mode. A modal identification technique from generalized buckling mode of FSM and its impact on the choice of basis, orthogonalization and normalization of vector spaces for local, distortional and global deformation spaces has been proposed by Li et al. [16]. The mode decomposition using constrained finite strip method (cFSM) and mode identification from constraint matrices has been extended for members with general boundary condition by Li and Schafer [17]. The cFSM procedure has been generalized by Ádány and Schafer [18,19] for closed cross-sections and cross-sections with open and closed parts for decomposition into mechanically meaningful subfields in addition to basic deformation modes.

Among the research related to decomposition of buckling modes from FEM, Casafont et al. [20] developed a procedure for decomposing buckling modes based on combined GBT and cFSM in which the finite element model is constrained to buckle in a particular mode. The procedure was extended to members with general boundary conditions by suitably varying the interpolation function for restraint matrix in the longitudinal direction [21]. Mode identification from shell finite element analysis using cFSM base functions for members subjected to axial compression and bending was also reported in literature [22,23]. The participation of global, distortional and local buckling mode in general modes from finite element analysis has been evaluated by Nedelcu and Cucu [24] using GBT cross-sectional deformation modes. The mode identification procedure using FSM base functions has also been applied to nonlinear analysis of shell finite element models [25]. Recently, a shell finite element was proposed by Ádány [26] for constrained shell finite element analysis.

In SFSM, buckling mode decomposition into combined global-distortional and local mode was done by Djafour et al. [27] using GBT basic assumptions based on procedure implemented in [15]. In the present investigation, base functions for buckling modes has been constructed based on cFSM and GBT principles and implemented in spline finite strip formulation of thin walled members under axial compression and flexure for various boundary conditions. Pure local, distortional and global buckling modes are evaluated by constraining the spline finite strip model to buckle in a particular mode.

2. Spline finite strip formulation

Spline finite strip method is well documented in the literature and hence it is not elaborated in this paper. However, for clarity, the formulation is briefly summarized. In this method, the thin walled prismatic member having length ‘a’ is discretized by ‘n’ nodal lines in transverse direction (x axis) and ‘m’ sections along longitudinal direction (y axis). Each section knot has four degree of freedom, ‘u’, ‘v’, ‘w’ and ‘θ_{xz}’ as shown in Fig. 1. The displacement function of the strip is expressed as the product of spline function in longitudinal direction and polynomial interpolation function in transverse direction.

$$\{d\} = [N][\phi]\{\delta\} \tag{1}$$

where $\{d\}$ is the vector of generalized displacements, $[N]$is the matrix of shape function in transverse direction, $[\phi]$ is the matrix of spline functions in longitudinal direction and $\{\delta\}$vector of displacements at section knots in the strip. The shape function in transverse direction is represented by Hermitian interpolation function for flexural displacements and Lagrangian interpolation function for membrane displacements.

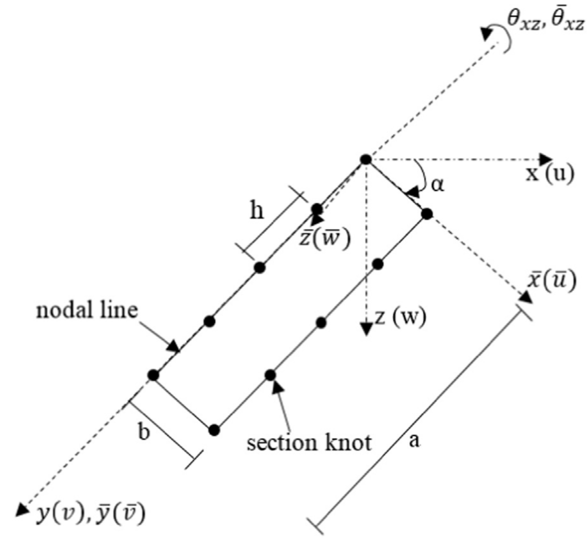

Fig. 1. Spline finite strip with coordinate axes.

Basic cubic spline (B3) having four sections has been adopted in the present analysis. A local B3 spline is a piecewise cubic polynomial which is twice differentiable (C2 continuous). A local B3 spline and spline series is shown in Fig. 2. A standard B3 spline function is defined in Eq. (2). Spline amendment schemes are implemented at ends for various boundary conditions. Two additional section knots (dummy knots) are introduced one at either end of the plate strip to completely define the amendment scheme. In this study, amendment scheme proposed by Fan [1] satisfying both geometric and natural boundary conditions has been implemented.

$$\phi_i(y) = \frac{1}{6h^3}\left\{\begin{array}{cc} (y-y_{i-2})^3 & y_{i-2} \le y \le y_{i-1} \\ h^3+3h^2(y-y_{i-1})+3h(y-y_{i-1})^2-3(y-y_{i-1})^3 & y_{i-1} \le y \le y_i \\ h^3+3h^2(y_{i+1}-y)+3h(y_{i+1}-y)^2-3(y_{i+1}-y)^3 & y_i \le y \le y_{i+1} \\ (y_{i+2}-y)^3 & y_{i+1} \le y \le y_{i+2} \\ 0 & otherwise \end{array}\right\} \tag{2}$$

The strain-displacement relation and stress-strain relation of thin orthotropic plate are established similar to the finite element analysis. Here $\{\varepsilon\}$ is the strain vector and $\{\sigma\}$ is the stress vector at various points, $[B]$ is the strain-displacement matrix and $[D]$ is the stress-strain matrix, the subscripts ‘b’ and ‘m’ corresponds to flexural and membrane

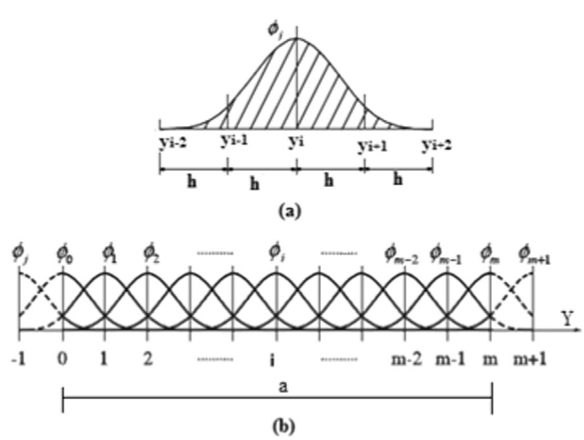

Fig. 2. (a) Local B3 spline (b) B3 spline series.