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Distortional buckling of thin-walled columns of closed quadratic cross-section

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ABSTRACT

The elastic stability of axially compressed column related to the cross-section distortion is investigated. Two kinds of closed quadratic cross-sections are taken into consideration with internal walls and without it. The governing differential equation is derived with aid of the principle of stationary total potential energy. The critical loads for the simply supported columns are found in an analytical form and compared with the FEM solution. Sufficient accuracy of the results is worth of noticing.

1. Introduction

In the classical Euler theory and the theory of restrained torsion of thin-walled bars it is assumed that the bar cross-section is non-deformable. In the context of engineering experience, this implies the need to use diaphragms in small intervals. If there are no diaphragms or the distance between them is large one should take into consideration the cross-section deformation of the bar. Up to date the local stability of walls in the frame of plate buckling analysis and the global stability of the bar has been well developed [1–5].

However, there are hardly a few papers dealing with the stability of bars including the deformability of the bar cross-section [6]. Particularly noteworthy is the article written by A. Chudzikiewicz [7], in which the possibility of stability loss due to the cross-section deformation is investigated.

The main topic of this paper is a detailed stability analysis of an axially compressed column of equal wall thickness with closed deformable quadratic cross-section with or without internal walls (Fig. 1a, b). In this case flexural buckling, torsional buckling and distortional buckling due to the cross-section deformation are independent of each other therefore in this paper only elastic distortional buckling analysis is considered. The one dimensional model of the column is taken into account and the governing differential equation is derived using the stationary energy theorem. Some numerical examples dealing with simply supported column with and without internal walls are presented and a comparative study of the analytical and the FEM results is provided as well.

2. Elastic energy of distortional buckling

Let us consider distortional stability of an axially compressed column stiffened by two diaphragms with freedom of warping at both ends (Fig. 1c). The governing differential equation of the problem is derived by means of the stationary total potential energy principle. The total potential energy Π is the sum of the elastic strain energy V stored in the deformed walls of bar together with the internal walls and the potential energy U of the applied loads.

2.1. The potential energy of cross-sectional distortion

The cross-section distortion is show in Fig. 1 and it is described by the distortion angle γ. The potential energy of cross-sectional distortions V_p due to the walls bending can be described as

– in the case of cross-sections without internal walls

$$V_p = \frac{1}{2}4\int_0^l\int_0^a \frac{M_p^2}{EJ_p}\,\mathrm{d}s\,\mathrm{d}z = \frac{24EJ_p}{a}\int_0^l \gamma^2\,\mathrm{d}z = \frac{1}{2}K_\gamma^{\square}\int_0^l \gamma^2\,\mathrm{d}z, \quad (1)$$

where M_p stands for the bending moment of external walls in transversal direction, E is the Young modulus, J_p is the moment of inertia of external wall cross-section in transversal direction, a is the height of cross-section and the factor $K_\gamma^{\square}$ is defined as

$$K_\gamma^{\square} \overset{def}{=} \frac{4E}{a}\delta_p^3, \quad (2)$$

where δ_p is the external walls thickness.

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Nomenclature

a –	Height of cross-section
n –	Number of half-wave of buckling mode
l –	Length of column
l_0 –	Characteristic length of column
r_0 –	Square of polar radius of gyration
s_p –	Distribution coefficient of axial load related to external wall
s_r –	Distribution coefficient of axial load related to internal wall
u –	Displacement of cross-section corner
x, y, z –	Cartesian coordinate system
A –	Area of cross-section
E –	Young's modulus
G –	Shear modulus
J_0 –	Polar moment of inertia
J_g –	Moment of inertia of external wall cross-section in longitudinal direction
J_p –	Moment of inertia of external wall cross-section in transversal direction
J_r –	Moment of inertia of internal wall cross-section in transversal direction
J_{sp} –	Free torsion moment of inertia of external wall
J_{sr} –	Free torsion moment of inertia of internal wall
K_g –	Factor of potential energy of bending
K_s –	Factor of potential energy of torsion
K_γ –	Factor of potential energy of distortion
M_p –	Bending moment of external walls in transversal direction
M_r –	Bending moment of internal walls in transversal direction
M_z –	Bending moment of external walls in longitudinal direction
$U_p{}^I$ –	Potential energy of compressive load due to bending
$U_p{}^{II}$ –	Potential energy of compressive load due to torsion
V –	Elastic strain energy
V_g –	Potential energy of elastic bending
V_p –	Potential energy of cross-section distortions
V_s –	Potential energy of torsion
P –	Compressive axial load
P_{cr} –	Critical buckling load
β_b –	Coefficient of bending effect
β_t –	Coefficient of torsion effect
γ –	Distortion angle
δ_p –	External walls thickness
δ_r –	Internal walls thickness
ν –	Poisson ratio
η –	Coefficient of characteristic length of column
σ_b –	Buckling stress
σ_{cr} –	Critical stress
σ_{crmin} –	Minimum critical stress
Π –	Total potential energy

Subscrips

$\square$ –	Cross-section without internal walls
$\boxplus$ –	Cross-section with internal walls

– In the case of cross-sections with internal walls

$$V_p = \frac{1}{2}\, 8 \int_0^l \int_0^{\frac{a}{2}} \frac{M_p^2}{EJ_p} \mathrm{d}s\, \mathrm{d}z + \frac{1}{2}\, 4 \int_0^l \int_0^{\frac{a}{2}} \frac{M_r^2}{EJ_r} \mathrm{d}s\, \mathrm{d}z = \frac{1}{2} K_\gamma^{\boxplus} \int_0^l \gamma^2\, \mathrm{d}z, \tag{3}$$

where M_r is the bending moment of internal walls in transversal direction, J_r is the moment of inertia of internal wall cross-section in transversal direction and the factor $K_\gamma^{\boxplus}$ is defined as

$$K_\gamma^{\boxplus} \overset{def}{=} \frac{4E}{a}\delta_p^3 \left(13 + \frac{\delta_r^3}{2\delta_p^3} - \frac{24\delta_p^3}{2\delta_p^3 + \delta_r^3}\right), \tag{4}$$

where δ_r is the internal walls thickness.

2.2. The bending energy of column in longitudinal direction

The elastic bending energy V_g of the column in longitudinal direction, in both cases under investigation is

$$V_g = \frac{1}{2}\, 4 \int_0^l \frac{M_z^2}{EJ_g} \mathrm{d}z = \frac{1}{2} K_g \int_0^l \gamma''^2\, \mathrm{d}z, \tag{5}$$

where M_z is the bending moment of external walls in longitudinal direction, J_g is the moment of inertia of external wall cross-section in longitudinal direction and the factor K_g is defined as

$$K_g \overset{def}{=} \frac{Ea^5}{12}\delta_p. \tag{6}$$

It should be noticing that the elastic bending of the internal walls in longitudinal direction is neglected.

2.3. The energy of torsion of the cross-section walls

In addition, the potential energy of torsion of the cross-section walls V_s is taken into account:

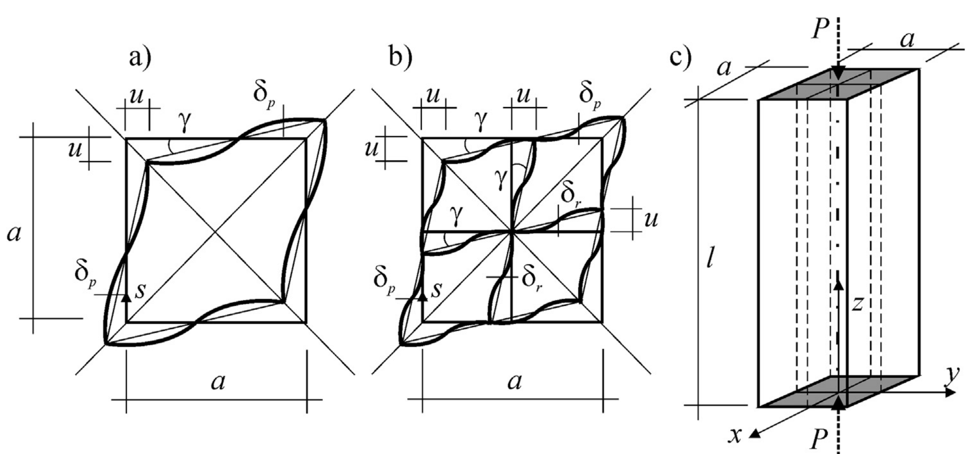

Fig. 1. Schematic diagram of the column and expected mode of distortional buckling (a) without and (b) with internal walls.