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Stability and free vibrations of the three layer beam with two binding layers

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ABSTRACT

The paper is devoted to the stability analysis of a simply supported three layer beam. The sandwich beam consists of two metal facings, a metal foam core and two binding layers between the faces and the core. In consequence, the beam is a five layer beam. The main goal of the study is to elaborate a mathematical model of the beam, analytical description and solution of the stability problem. The beam is subjected to an axial compression and, in particular, a pulsating compression. The nonlinear hypothesis of deformation of the cross section of the beam is formulated. Based on the Hamilton's principle the system of four stability equations is derived. The system is reduced to one equation of motion (Mathieu's equation) serving as a basis for determining the critical loads, free vibrations and unstable regions. The influence of the binding layers is considered. The results of solutions of the vibration problem analysis are shown in tables and figures. The analytical model is verified numerically with the use of Finite Element Analysis.

1. Introduction

Sandwich structures are widely applied since the mid of 20th century, for example in aerospace, automotive, rail and shipbuilding industry. These structures are characterized by high stiffness with regard to their mass. Ashby et al. [1] described the mechanical properties of metal foams. Banhart [2] provided a comprehensive description of various manufacturing processes of metal foams and porous metallic structures. Kubiak [3] studied buckling and postbuckling behavior of thin plates and thin-walled structures with flat walls, subjected to static and dynamic load. Ventsel and Krauthammer [4] presented principles of thin plate and shell theories, emphasized novel analytical and numerical methods for solving linear and nonlinear plate and shell problems. Belica et al. [5] presented a nonlinear approach to dynamic stability of an isotropic circular cylindrical shell made of metal foam and subjected to combined loads. Jasion [6], Jasion and Magnucki [7,8] studied analytically, numerically and experimentally the global and local buckling-wrinkling of the face sheets of sandwich beams. Mania [9] analyzed the dynamic response of FGM thin plate structures subjected to combined loads. Małachowski et al. [10] presented the experimental investigations and numerical modelling of closed-cell aluminium alloy foam (Alporas). Jasion and Magnucki [7] analyzed the local buckling problem of sandwich beams under pure bending. Magnucka-Blandzi [11] carried out a theoretical study on dynamic stability of a metal foam circular plate. Magnucka-Blandzi and Magnucki [12] optimized the sandwich beam with metal foam core under strength and stability constrains. Magnucki et al. [13] studied three-layer beams with corrugated core subjected to compression and four point bending. Magnucki et al. [14,15] and Smyczynski and Magnucka-Blandzi [16] presented the strength analysis of a simply supported five layer sandwich beams with a metal foam core. Smyczynski and Magnucka-Blandzi [17] analyzed the stability of a five layer sandwich beam with the use of broken line hypothesis of the deformation of a flat cross section of the beam. Kim et al. [18] studied a dynamic stability behavior of the shear-flexible composite beams subjected to the nonconservative force based on finite element model using Hermitian beam elements. Magnucka-Blandzi [19] compared the results of vibration problem of a sandwich beams for the three different modified Timoshenko hypotheses of deformation. Grygorowicz et al. [20] analytically and numerically studied elastic buckling of a three-layered beam with variable mechanical properties of the core. Pawlus [21,22] presented the computational results of critical loads calculations of annular three-layered plates with a soft core. Yang et al. [23] analyzed the dynamic stability of composite laminated beams with delaminations. Loja et al. [24] considered the use of various shear deformation theories to formulate different layerwise models, implemented through kriging-based finite elements. They solved dynamic problem of soft core sandwich beams in frequency domain. Mohanty et al. [25] presented the evaluation of static and dynamic behavior of functionally graded Timoshenko beams. Smith et al. [26] and Szyniszewski et al. [27,28] characterized mechanical properties of hollow sphere steel foam. They provided and verified a new design

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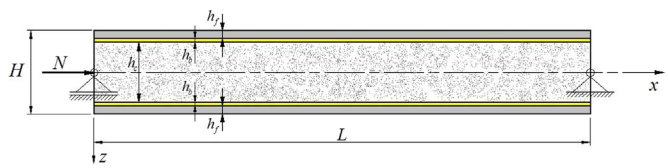

Fig. 1. Scheme of the five layer beam subjected to an axial force.

method for the in-plane compressive strength of steel sandwich panels composed of steel face sheets and foamed steel cores.

The present paper is devoted to stability analysis of a simply supported sandwich beam, which consists of five layers: two thin facings (aluminium sheets) of thickness h_f, one core (aluminium foam) of thickness h_c and two thin binding layers (e.g. glue) of thickness h_b. Mechanical properties of each layer are different and depend on their material. The beam has the length L, the width b and the depth H. The beam carries a compressive axial load N (Fig. 1) varying in time and assumed in the following form:

$$N(t) = N_0 + N_a \cos(\theta t), \tag{1}$$

where

N_0– average value of the load,
N_a– amplitude of the load,
θ, t– frequency and the time dependent compressive load $N(t)$.

2. Nonlinear hypothesis of deformation of a flat cross section of the beam

The field of displacement for the flat cross section of the five layer beam is presented in Fig. 2. Assuming the nonlinear hypothesis the shear effect is taken into account.

The longitudinal displacements are formulated as follows:

1. for the upper facing $-(1/2 + x_1 + x_2) \le \zeta \le -(1/2 + x_1)$

$$u(x, \zeta, t) = -h_c\left[\zeta\frac{\partial w}{\partial x} + \psi_1(x, t)\right], \tag{2}$$

2. for the upper binding layer $-(1/2 + x_1) \le \zeta \le -1/2$

$$u(x, \zeta, t) = -h_c\left\{\zeta\frac{\partial w}{\partial x} + \psi_2(x, t) - \frac{1}{x_1}\left(\zeta + \frac{1}{2}\right)[\psi_1(x, t) - \psi_2(x, t)]\right\}, \tag{3}$$

3. for the core $-1/2 \le \zeta \le 1/2$

$$u(x, \zeta, t) = -h_c\left\{\zeta\left[\frac{\partial w}{\partial x} - 2\psi_2(x, t)\right] + \frac{1}{2\pi}\psi_3(x, t)\sin(2\pi\zeta)\right\}, \tag{4}$$

4. for the lower binding layer $1/2 \le \zeta \le 1/2 + x_1$

$$u(x, \zeta, t) = -h_c\left\{\zeta\frac{\partial w}{\partial x} - \psi_2(x, t) - \frac{1}{x_1}\left(\zeta - \frac{1}{2}\right)[\psi_1(x, t) - \psi_2(x, t)]\right\}, \tag{5}$$

5. for the lower facing$1/2 + x_1 \le \zeta \le 1/2 + x_1 + x_2$

$$u(x, \zeta, t) = -h_c\left[\zeta\frac{\partial w}{\partial x} - \psi_1(x, t)\right], \tag{6}$$

where
$x_1 = h_b/h_c$, $x_2 = h_f/h_c$– dimensionless parameters,
$\zeta = z/h_c$– dimensionless coordinate,
$\psi_1(x, t) = u_1(x, t)/h_c$, $\psi_2(x, t) = u_2(x, t)/h_c$, $\psi_3(x, t)$– dimensionless functions of displacement, which determine the field of displacements.

If $\psi_3 \equiv 0$ the proposed nonlinear hypothesis becomes the broken line hypothesis. So the assumed hypothesis is a generalization of the classical one described in [14,15,17].

Strains of the layers of the five layer beam are defined by the following geometric relations:

$$\varepsilon_x = \frac{\partial u}{\partial x},\ \gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z},$$

so

1. for the upper facing $-(1/2 + x_1 + x_2) \le \zeta \le -(1/2 + x_1)$

$$\varepsilon_x = -h_c\left(\zeta\frac{\partial^2 w}{\partial x^2} + \frac{\partial \psi_1}{\partial x}\right),\ \gamma_{xz} = 0, \tag{7}$$

2. for the upper binding layer $-(1/2 + x_1) \le \zeta \le -1/2$

$$\varepsilon_x = -h_c\left[\zeta\frac{\partial^2 w}{\partial x^2} + \frac{\partial \psi_2}{\partial x} - \frac{1}{x_1}(\zeta + \tfrac{1}{2})(\frac{\partial \psi_1}{\partial x} - \frac{\partial \psi_2}{\partial x})\right],\quad \gamma_{xz} = \frac{1}{x_1}(\psi_1 - \psi_2), \tag{8}$$

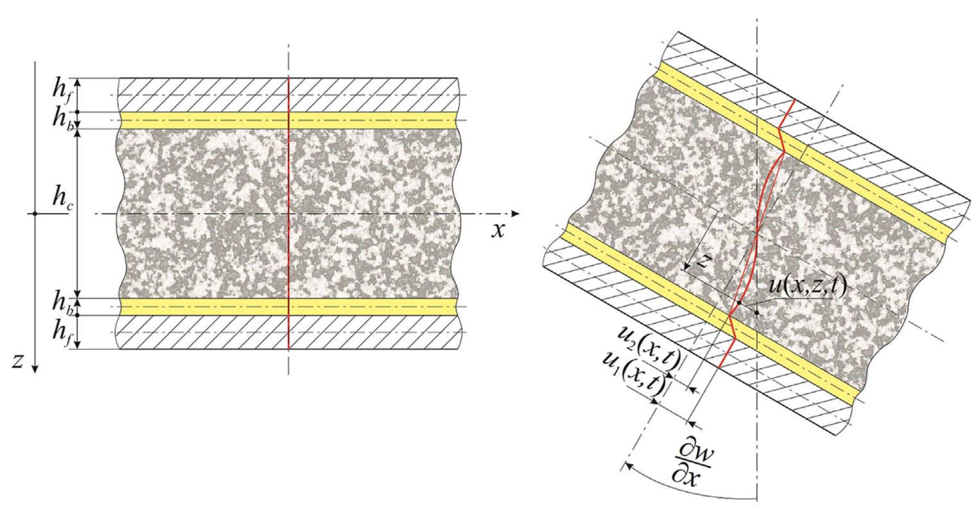

Fig. 2. The field of displacement – a nonlinear hypothesis.

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