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Nonlinear bending and vibration analyses of quadrilateral composite plates

Amit Kumar, M.K. Singha*, Vikrant Tiwari

Department of Applied Mechanics, Indian Institute of Technology Delhi, New Delhi 110016, India

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ABSTRACT

Linear and nonlinear analyses of shear deformable thin and thick arbitrary straight-sided quadrilateral plates are reported here using smoothed finite element technique. The Reissner-Mindlin plates are discretized with quadrilateral background cells. Then membrane and bending stiffness matrices of background quadrilateral cells are evaluated using *edge-based* smoothed finite element method(ES-FEM). The shear stiffness matrix is calculated based on "*smoothed shear strain approach*" and the performance is compared with "*edge-consistent four-node quadrilateral finite element*". The convergence, accuracy and sensitivity to mesh distortion of the present quadrilateral element is examined. Thereafter, the nonlinear bending and vibration analyses of trapezoidal and arbitrary straight-sided quadrilateral composite plates are presented for which only limited analytical results are available in the literature.

1. Introduction

Non-rectangular plates, like skew, trapezoidal and quadrilateral plates find wide application in civil, mechanical and aerospace industry. Linear bending and vibration analyses of such quadrilateral plates have been attempted by several investigators using various analytical methods [1–16]. Liew [1] employed pb2-Ritz method, while, Saadatpour and Azhari [2] used Galerkin method for the linear static analysis of thin trapezoidal plates. Free vibration problem of thin trapezoidal plates are solved using Galerkin's method [3–5], superposition method [6], differential quadrature method [7,8], Rayleigh-Ritz method [9] and spline finite strip method [10]. Bending, buckling and vibration behaviour of thin quadrilateral plates are investigated by Civalek [11] using discrete singular convolution method. Differential quadrature method [12,13], Rayleigh-Ritz method [14] and discrete singular convolution method [15] was also employed to study moderately thick trapezoidal plates. However, the above analytical works deals with the bending / vibration behaviour of thin / thick trapezoidal plates using linear structural theory. Leung and Zhu [16] attempted geometrically nonlinear vibration of trapezoidal plates using trapezoidal hierarchical finite element method. Recently, Shufrin et al. [17] employed multi-term extended Kantorovich method for the geometrically nonlinear bending analysis of thin trapezoidal plates.

Numerical methods, such as the finite element method has been widely used for nonlinear static and dynamic analyses of plates and shells with complicated loading and boundary conditions. However, use of numerical techniques for the analysis of trapezoidal / quadrilateral plates is scarce in the literature, even if skew plates are widely investigated using the finite element method [18,19]. Orris et al. [20] proposed a quadrilateral element by combining triangular elements to analyze thin trapezoidal plates, while Barik and Mukhopadhyay [21] attempted free vibration analysis of thin trapezoidal plates using isoparametric finite element. In the case of non-rectangular domain, the relationship between the isoparametric coordinates (ξ, η) and Cartesian coordinates (x, y) is nonlinear and it is well know that the accuracy of the commonly used "*isoparametric elements*" reduces with mesh distortion. To overcome the above difficulty, several *Mindlin-Reissner quadrilateral plate bending elements* are proposed in the literature [22] and the quadrilateral elements are tested mostly for rectangular / skew plates. After critical review of literature on this subject, Cen and Shang [22] observed that a promising distortion-insensitive finite element for quadrilateral plate is missing. Therefore, only limited study on the bending / buckling / vibration behaviour of arbitrary straight-sided quadrilateral plates are available in the literature [23–29] using different analytical approaches or mesh-free methods.

Smoothed finite element method (SFEM) proposed by Liu et al. [30] appears to be a promising numerical method for modelling non-rectangular domain. Subsequently, the cell based smoothed finite element method has been employed to develop triangular [31,32] and four-node quadrilateral [33,34] Reissner–Mindlin plate bending elements. The curvature smoothing technique has been used for estimating the bending strain. The shear locking phenomenon of Reissner–Mindlin plates was eliminated by the discrete shear gap (DSG) technique for triangular elements [31,32] and mixed interpola-

* Corresponding author.
E-mail address: maloy@am.iitd.ernet.in (M.K. Singha).

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tion concept (MITC) for quadrilateral elements [33,34]. Wu and Wang [35] discussed about the shear stress oscillation of MITC4 element for distorted mesh and proposed an enhanced CS-FEM based on *smoothed curvature and smoothed shear strain* (spanned over the adjacent element). The above elements are found to perform well for rectangular, skew and circular plates.

Subsequently, Liu et al. [36] proposed an edge-based smoothed finite element method (ES-FEM) for solid mechanics problems. The authors employed triangular elements to analyze static and dynamic problems of solid mechanics and concluded that the proposed ES-FEM is "*super-convergent and ultra-accurate*". Edge-based SFEM with triangular elements has also been extended for the analysis of Mindlin–Reissner plates by Cui et al. [37] and Nguyen-Xuan et al. [38]. The membrane and bending stiffness matrices were evaluated using edge-based smoothing domains, while stabilized discrete shear gap (DSG) technique for triangular elements was employed for the shear stiffness matrix and this combined method was named as "*edge-based smoothed stabilized discrete shear gap method* (ES-DSG3)" by Nguyen-Xuan et al. [38]. Phung-Van et al. [39] extended the ES-DSG3 method for the analysis of composite and sandwich plates using layer-wise theory.

It is observed from the literature that the cell-based and edge-based smoothed finite element methods [30,36] have been mostly used with triangular / rectangular elements to analyze shear deformable plates. However, to the best of the author's knowledge, the efficiency of ES-FEM with quadrilateral elements has not been explored yet, possibly due to the difficulty of interpolating locking-free shear strain in Reissner–Mindlin quadrilateral elements. Moreover, only limited study is reported on the geometrically nonlinear analysis of Reissner–Mindlin plates using the smoothed finite element method [40,41].

The purpose of the present work is to formulate distortion insensitive quadrilateral elements for Reissner-Mindlin plates using edge-based (ES-FEM) smoothed finite element method. The membrane strains and curvatures inside the smoothing domains are evaluated using smoothing techniques [36]. The shear strain along the tangential direction of an inclined edge is assumed to be constant [42,43] and evaluated using the displacements and rotations of the two nodes connecting the inclined edge. The relative shear rotations of the nodes are evaluated using the tangential shear strains of the four edges and the smoothed shear strain field is proposed. The performances of the developed elements are tested for shear deformable thin and thick trapezoidal and quadrilateral plates for which only limited analytical works are available in the literature. Thereafter, the present numerical techniques are extended for geometrically nonlinear bending and free vibration analyses of quadrilateral composite plates for which results are scarce in the literature.

2. Smoothed finite element formulation

The purpose of the present study is to analyze moderately thick arbitrary straight-sided quadrilateral plates including the effect of transverse shear deformation. Accordingly, the first-order shear deformation theory (Reissner–Mindlin plate theory) of plate is considered here. The displacement components (u_0, v_0, w) at a point (x, y, z) of a quadrilateral plate may be written as:

$$u(x, y, z) = u_0(x, y) + z\theta_x(x, y)$$
$$v(x, y, z) = v_0(x, y) + z\theta_y(x, y);$$
$$w(x, y, z) = w_0(x, y) \tag{1}$$

Here, u_0, v_0 and w are the mid-surface displacements; θ_x and θ_y are independent rotations of the normal to the mid-surface in xz and yz planes, respectively.

Following von Kármán strain-displacement assumption, the in-plane (ε_{xx}, ε_{yy} and ε_{xy}) strains may be written as:

$$\{\varepsilon\} = \{\varepsilon_{xx}\ \ \varepsilon_{yy}\ \ \varepsilon_{xy}\}^T = \{\varepsilon_m^L\} + \{\varepsilon_m^{NL}\} + z\,\{\varepsilon_b\} \tag{2}$$

Here, $\{\varepsilon_m^L\}$ are $\{\varepsilon_m^{NL}\}$ the linear and nonlinear components of the membrane strains; $\{\varepsilon_b\}$ is the curvatures. The strain components are given by

$$\varepsilon_m^L = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{Bmatrix};$$
$$\varepsilon_m^{NL} = \frac{1}{2}\begin{Bmatrix} (w_{,x})^2 \\ (w_{,y})^2 \\ (w_{,x}w_{,y}) \end{Bmatrix} \text{ and}$$
$$\varepsilon_b = \begin{Bmatrix} \frac{\partial \theta_x}{\partial x} \\ \frac{\partial \theta_y}{\partial y} \\ \frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x} \end{Bmatrix} \tag{3}$$

Here, $w_{,x}$ and $w_{,y}$ represent partial derivatives of "w" with respect to "x" and "y" respectively. The shear strains $\{\varepsilon_s\}$ may be written as:

$$\{\varepsilon_s\} = \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} \theta_x + w_{,x} \\ \theta_y + w_{,y} \end{Bmatrix} \tag{4}$$

The internal strain energy ($U = U_m + U_b + U_s$) of the plate may be written as

$$U_m(\delta) = \frac{1}{2}\int_A [\{\varepsilon_m^L + \varepsilon_m^{NL}\}^T[A]\,\{\varepsilon_m^L + \varepsilon_m^{NL}\} + \{\varepsilon_m^L + \varepsilon_m^{NL}\}^T[B]\,\{\varepsilon_b\} + \{\varepsilon_b\}^T[B]\,\{\varepsilon_m^L + \varepsilon_m^{NL}\}]\,dA \tag{5a}$$

$$U_b(\delta) = \frac{1}{2}\int_A \{\varepsilon_b\}^T[D]\,\{\varepsilon_b\}\,dA \tag{5b}$$

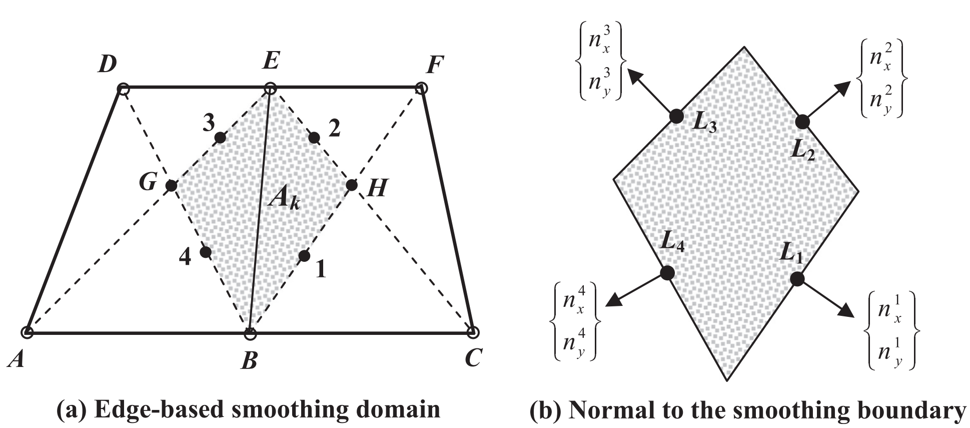

Fig. 1. Schematic representation of the edge-based smoothing domain of quadrilateral elements (field nodes are encircled).