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Post-buckling analysis of variable-angle tow composite plates using Koiter's approach and the finite element method

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ABSTRACT

The design of lightweight structures is often driven by buckling phenomena. Increasing demands for fuel-efficient aircraft structures makes post-buckled designs attractive from a structural weight perspective. However, the need for reliable and efficient design tools that accurately model the emerging nonlinear post-buckled landscape, potentially one containing multiple branches, remains. With this aim, a previously derived flat shell element, MISS-4, is extended to the geometrically nonlinear analysis of variable-angle tow (VAT) composite plates using Koiter's asymptotic approach. The curvilinear fiber paths in VAT lamina open the design space for tailoring the buckling and post-buckling capability of plates and shells. A finite element implementation of Koiter's asymptotic approach allows the pre-critical and post-critical behavior of slender elastic structures to be evaluated in a computationally efficient manner. Its implementation uses a fourth-order expansion of the strain energy, and requires both the structural modeling and finite element discretization procedures to be, at least, of fourth order. The corotational approach adopted in the MISS-4 element readily fulfills this requirement by starting from a linear finite element discretization. VAT plates with prismatic fiber variations and different loading conditions are analyzed using the MISS-4 element and numerical results of the post-buckling paths are presented. The computed equilibrium paths are compared to benchmark results using the commercial finite element package ABAQUS, and strong asymptotic solutions of the differential equations. The results document the good accuracy and reliability of the proposed modeling approach, and also highlight the importance of multi-modal analysis when multiple buckling modes coincide as is the case in long plates, shells and other optimized thin-walled structures.

1. Introduction

In the analysis of slender elastic structures, the numerical implementation of Koiter's asymptotic method [1] allows a reliable evaluation of the post-buckling behavior. The method, its implementation and application are under investigation to this day [2–7]. The advantages of Koiter's asymptotic approach are apparent when multi-modal buckling interactions are to be accounted for. Furthermore, the sensitivity to imperfections, and therefore a realistic evaluation of the load carrying capacity of the structure, can be studied in a computationally efficient manner by combining Koiter's asymptotic approach with stochastic Monte Carlo simulations [8–12].

A finite element implementation of Koiter's asymptotic method was developed by Casciaro et al. [13,14]. Initially, this method was applied to planar beam frames [15,16] and simple plate assemblies [17]. The use of a corotational formulation [18,19] allowed the simple extension of the underlying linear model to the geometrically nonlinear regime and permitted the straightforward computation of high-order strain energy derivatives. As a result, the analysis could be extended to spatial beam frames [20] and general shell structures [21]. Recently, the formulation has been extended to laminate composite shell structures [9], particularly to cylindrical shells [22] and folded plate structures [10], and to cold formed steel structures [23]. By coupling the asymptotic method with a Monte Carlo engine, the effect of random imperfections on the first critical load of cylindrical shells in compression [22], and the erosion [24–26] of critical loads of cold formed sections have been investigated [23]. Recently, the method has been validated experimentally for composite beams [27] confirming the good accuracy and reliability of the approach.

Here, a general finite element for the pre-buckling, buckling and post-buckling analysis of variable-angle tow (VAT) plates using Koiter's asymptotic approach is proposed. Due to the increased design space for locally tailoring laminate stiffness, VAT composites are a promising technology for further improving the structural efficiency of engineer-

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ing structures. In these variable stiffness structures, the fiber tows within a layer are not restricted to straight trajectories but can describe curvilinear paths. Numerous works have shown that tailoring the in-plane stiffness over the plate planform allows pre-buckling stresses to be redistributed to supported regions, thereby increasing the first critical buckling load [28–35].

Wu et al. [36] showed that the fiber orientations of flat VAT laminates can be tailored to reduce the stiffness knock-down in the post-critical regime compared to straight-fiber laminates. Furthermore, the optimal fiber paths for increasing buckling load also reduce the out-of-plane post-buckling displacements [37]. An interesting application of variable-stiffness composites is the reduction of the imperfection sensitivity of shell structures. It is well known in the engineering community that cylindrical shells are prone to collapse beyond the first critical load, and as a result of this instability, shell structures are extremely sensitive to geometric and loading imperfections. White and Weaver [38] showed that this imperfection sensitivity can be effectively eliminated by tailoring the fiber paths across the surface of cylindrical shells. Hence, stable plate-like post-buckling responses in cylindrical shells were documented for the first time.

Due to its modeling versatility and numerical robustness, most modeling work on the buckling of VAT structures has focused on using the finite element method (FEM) [30,39–42]. At the same time, the pseudo-spectral differential quadrature method (DQM) has been shown to be a fast, accurate and computationally efficient technique for solving the variable-coefficient higher-order differential equations for buckling [33,43] and post-buckling [35,44] of VAT plates and cylindrical shells [45,46].

White et al. [45] were the first to implement Koiter's asymptotic approach within a DQM framework. The asymptotic differential equations for the pre-buckling, buckling and initial post-buckling problem of constant stiffness curved panels were solved using the generalized DQM, while the orthogonality condition of the buckling and initial post-buckling mode shapes was enforced as an additional constraint equation using an integral quadrature approach. The over-determined system was then solved using the Moore-Penrose generalized matrix inverse operation. This general approach was used in the optimization study of imperfection-insensitive cylindrical shells mentioned above, and by Groh and Weaver [46] in a minimum-mass optimization study of VAT wing panels with static failure constraints. Finally, Raju et al. [47] extended the numerical scheme beyond the initial post-buckling regime, and used the approach as the basis for an optimization scheme to minimize the end-shortening strain in the post-buckling regime.

The starting point of the present implementation of Koiter's asymptotic approach in the finite element method is the 3D shell element MISS-4 [48–50]. The use of a corotational reference frame allows the extension of the geometrically linear finite element into a nonlinear context. The fundamental equations of Koiter's asymptotic method are presented in Section 2. Derivations of the linear shell element MISS-4 are discussed in Section 3, and its extension to the geometrically nonlinear regime presented in Section 4. In Section 5, a number of different test cases with different VAT stacking sequences and loading conditions are presented and comparisons are made with benchmark models. In particular, the results have been compared with the Riks path-following algorithm [51] in the commercial code ABAQUS [52] and with the implementation of Koiter's approach in DQM [45]. Furthermore, this section highlights the importance of multi-modal expansions in Koiter's approach. Finally, conclusions are drawn in Section 6.

2. Nonlinear analysis of slender elastic structures

The starting point of Koiter's asymptotic approach is the total potential energy functional $\Pi[u]$, where u are the configuration variables (i.e. displacements/stresses). In particular, we have

$$\Pi[u] = \Phi[u] - \lambda p u \tag{1}$$

where $\Phi[u]$ is the strain energy, λ the load control parameter and p the applied load. The solution to the problem is given by the stationarity condition of the total potential energy

$$\Pi[u] = \text{stat.} \tag{2}$$

which means that the first variation of the total potential energy must vanish. Hence,

$$\Pi'[u]\delta u = \Phi'[u]\delta u - \lambda p \delta u = 0 \quad \forall\ \delta u \tag{3}$$

where the prime denotes the Frèchet derivative with respect to configuration variable u, and the equilibrium Eq. (3) is generally nonlinear. Using the finite element method, Eq. (3) can be rewritten as

$$\delta\boldsymbol{u}^T\boldsymbol{r}[\boldsymbol{u},\lambda] = 0 \quad \forall\ \delta\boldsymbol{u} \quad \boldsymbol{r}[\boldsymbol{u},\lambda] = (\boldsymbol{s}[\boldsymbol{u}] - \lambda\boldsymbol{p}) \tag{4}$$

with $u = \mathcal{L}\boldsymbol{u}$ and $p = \mathcal{L}\boldsymbol{p}$, where $\mathcal{L}$ is a suitable interpolation operator, and $\boldsymbol{s}[\boldsymbol{u}]$ and $\boldsymbol{p}$ represent the internal and external load vectors, respectively. The solution of Eq. (4), and therefore the equilibrium path, can be obtained either by means of a path-following approach or an asymptotic approach.

The path-following [51] approach is widely used as a solution scheme because of its general applicability to a wide array of nonlinear systems. Disadvantages include the computational cost, which is directly related to the number of variables in the FEM discretization (i.e. the dimension $\boldsymbol{u}$); the need to perform separate analyses in the case of small modifications (i.e. imperfections) of the load and/or geometry; and difficulties associated with following post-critical branches at coincident, or near-coincident, critical loads.

On the other hand, in the asymptotic analysis [1], the equilibrium path is obtained in an approximate fashion through an asymptotic expansion of parameters ξ_i for $i = 0 \ldots m$. Starting from a known expansion point $\{\boldsymbol{u}_0, \lambda_0\}$, we have:

$$\begin{aligned}\boldsymbol{u} &= \boldsymbol{u}_0 + \sum_{i=0}^{n}\xi_i\dot{\boldsymbol{u}}_i + \frac{1}{2}\sum_{i,j=0}^{m}\xi_i\xi_j\ddot{\boldsymbol{u}}_{ij} + \cdots + O(\xi_i^k),\\ \lambda &= \lambda_0 + \sum_{i=0}^{n}\xi_i\dot{\lambda}_i + \frac{1}{2}\sum_{i,j=0}^{m}\xi_i\xi_j\ddot{\lambda}_{ij} + \cdots + O(\xi_i^k)\end{aligned} \tag{5}$$

where the superimposed $(\dot{\cdot})$ denotes differentiation with respect to ξ. By invoking the fundamental lemma of the calculus of variations, the nonlinear system of Eq. (4) can be rewritten as

$$\boldsymbol{r}[\boldsymbol{\xi},\lambda] = \boldsymbol{0} \tag{6}$$

with vector $\boldsymbol{\xi}$ collecting the ξ_i expansion parameters. The nonlinear system Eq. (6) is generally defined by a reduced number of variables. In practical contexts the expansion order m is of order 10^1. Once all unknowns in Eq. (5) are determined for the so-called 'perfect structure', the solution for deviations in the assumed load and geometrical imperfections only require the solution of Eq. (6). Thus, the reduced order of the asymptotic method allows the straightforward computation of thousands of imperfections at low computational cost.

2.1. Koiter's approach fundamental equations: Casciaro's quadratic algorithm

In the following, Koiter's asymptotic approach as proposed by Casciaro [13,14] in the context of the FEM is employed. This implementation is also know as the *quadratic algorithm* [13] and has been developed over the last thirty years.

The analysis procedure can be summarized as follows:

– First, the *fundamental path* (pre-critical path) is assumed to be a linear combination

$$\boldsymbol{u}^f[\lambda] = \boldsymbol{u}_0 + \lambda\hat{\boldsymbol{u}} \tag{7}$$

where $\boldsymbol{u}_0$ is an initial known configuration, and $\hat{\boldsymbol{u}} = d\boldsymbol{u}/d\lambda$ is computed