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# Buckling of piecewise member composed of steel and high-strength materials in axial compression

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ABSTRACT

The elastic buckling load of an axially compressed piecewise member was derived under various ideal boundary conditions based on the differential element method; then, the result was simplified for a symmetrical member. The theoretical results were found to be consistent with existing formulas for the elastic buckling load of uniform members. Then, the elasto-plastic buckling of an axially compressed piecewise member composed of steel and elastic materials was investigated and found to be quite different from that of a pure steel member, and a theoretical analysis approach was developed. In elasto-plastic buckling analysis, the load-bearing capacity of the steel section in the ultimate state is determined considering initial imperfections and steel hardening. Furthermore, the symmetric method for obtaining the load-bearing capacity of this member was obtained based on the ultimate state of the critical section, which is simple and reparative for design. The theoretical results of the elasto-plastic buckling analysis were consistent with experimental results, therein obtaining an average error of 4%.

## 1. Introduction

### 1.1. Brief introduction to piecewise members

Piecewise members incorporating steel are commonly seen in engineering, mostly in buckling restrained braces (BRBs) and strengthened compressive members. BRBs have un-strengthened segments to provide easy connection to the main structure (Fig. 1(a)), as has been widely used in civil engineering as energy dissipation components [1,2]. For compressive steel member strengthening, the most widely used methods, including welding additional metal, bonding steel plates [3], pasting FRPs (fiber reinforced polymers) [4,5], using RC enclosures or fillings [6], applying FRP wrappings and internal filling reinforcements [7], and other complicated forms [8,9], use the un-strengthened ends of the existing steel member to avoid complicated configurations in joint regions. Some examples of FRP wrappings and internal filling reinforcements are illustrated in Fig. 1(b) and (c). Compared with other strengthening methods, such examples represent an ideal form of an exterior restraint system. Previous studies have demonstrated the effectiveness of this exterior restraint system, with the bearing capacity of some members approximating $f_u A_s$ [10,11]. The reasons for this are as follows. Assuming that the plane cross-section assumption is satisfied, the strain of the exterior restraint system at the edge of the section will be greater than that of the steel core in bending. If a linear elastic material in the exterior restraint system has a smaller yield strain or ultimate strength, it may become damaged before yielding of the core. Therefore, the exterior restraint system should have considerable strength to avoid this situation. However, traditional strengthening methods, such as welding or sticking additional metal plates and using reinforced concrete enclosures, use steel in the exterior restraint system. In this way, the strength of the steel core is not sufficiently utilized, and the load may suddenly fall after the steel yields. FRP is linear elastic, and the ultimate strain can be higher than 25,000 με (10 times the yield strain of steel), making it suitable as an exterior restraint system.

These two types of members, BRBs and strengthened compressive members, can be analyzed using the mechanical model shown in Fig. 1(d) and (e). This member is divided into the strengthened middle segment and the un-strengthened segments at the ends axially. The strengthened section is composed of the steel core and the exterior restraint system, between which exists an interface. The compression capacity of this member is closely related to the bending stiffness, axial stiffness, length of the exterior restraint system, and interface properties.

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**Nomenclature**

| | | | |
|---|---|---|---|
| $A$ | cross-sectional area | $P_e$ | elastic buckling load |
| $E$ | elastic modulus | $P_{ep}$ | elasto-plastic buckling load |
| $f_y$ | yield stress of the steel | $\overline{P_s}$ | allowable axial force of the steel core |
| $f_u$ | ultimate stress of the steel | $Q$ | shear force |
| $I$ | moment of inertia | $q_h$ | horizontal distributed forces |
| $L$ | full length of the member | $q_v$ | vertical distributed forces |
| $l_1$, $l_3$ | length of the un-strengthened segments | $x$ | axial coordinate of the specimen |
| $l_2$ | length of the strengthened segment | $y$ | lateral coordinate of the specimen |
| $M$ | bending moment | $\alpha$ | length ratio |
| $\overline{M_s}$ | allowable bending moment of the steel core | $\beta$ | bending stiffness ratio |
| $P$ | axial force | $\eta$ | axial stiffness ratio |
| | | $\mu$ | effective length factor |
| | | $v_0$ | maximum initial deflection |

### 1.2. Analysis difficulties

There are four difficulties faced in the buckling analysis of this member.

(1) The steel core and the exterior restraint system are usually characterized by different mechanical properties. If the exterior restraint system is elastic, it never yields as does the core steel; if the steel core and the exterior restraint system are both elasto-plastic materials, these two parts will generally not yield simultaneously. The complicated nonlinear mechanical performance of the piecewise member therefore exhibits stages.
(2) If the exterior restraint system has a high strength, the axial load on the piecewise member can continue increasing after the steel yields. Therefore, the steel stress could reach $f_u$, making it necessary to consider the steel hardening.
(3) The piecewise member is geometrically non-uniform.
(4) The influence of the interface property on the load bearing capacity of the piecewise member is unknown.

The first two difficulties are related to material nonlinearity after yielding and buckling. However, these issues, especially the steel hardening, are not sufficiently considered in current designs. In the design codes for the strengthening of steel members [3], the average stress of the steel cannot exceed the nominal yield strength $f_y$; thus, the steel is always in the elastic state. In a BRB's design, global buckling of the steel core is prevented before the axial load reaches $f_y A_s$; therefore, it is also an elastic analysis when examining buckling [12]. However, in the compression experiments on strengthened steel members with high-strength materials, the load bearing capacities of piecewise members can exceed $f_y A_s$, even approaching $f_u A_s$ [10,11]. The calculation of the elasto-plastic load bearing capacity of piecewise members is not only the foundation of a safe and economical design of strengthening steel members and BRBs but also the expansion of traditional stability theory. Furthermore, the load bearing capacity after steel yielding and member buckling, which is related to collapse prevention of the whole structure under extreme disaster conditions, should be investigated [13].

Concerning the third difficulty, studies on the stability bearing capacities of piecewise members have been performed. Arbabi et al. [14] and Coşkun et al. [15] derived the elastic Euler buckling load for piecewise members. Liu et al. [16] conducted compression experiments for FRP strengthening of intermediate weakened steel members and derived and verified the Euler buckling load. Krauberge [17] calculated the eigenvalue buckling load for different forms of partially weakened bars. However, these studies did not focus on the elasto-plastic load bearing capacity of piecewise members.

Finally, few studies have focused on the interface properties of a piecewise member, as shown in Fig. 1(d). There are two assumed limit conditions. In most strengthening analyses, the steel core and the exterior restraint system are fully combined. In BRBs, presumably, steel members and the restraint system do not transmit axial forces. However, in experiments involving FRP strengthening of steel members, complex stress distributions and slippage have been observed at the interface [6,7]. Therefore, the influence of interface properties on the load bearing capacity of piecewise members needs to be investigated.

Based on the above, this paper focuses on the elastic and elasto-plastic load of compressed piecewise members composed of steel and elastic materials. Theoretical approaches to the buckling analysis of compressed piecewise members are compared in Section 2. Elastic analysis of the load bearing capacity of piecewise members under various ideal boundary conditions is conducted and discussed in Section 3. In Section 4, a theoretical analysis approach is developed. The *P-M* correlation curve of strengthened steel sections in the ultimate state considering steel hardening is discussed, and the elasto-plastic buckling load and behaviors after steel yielding of the piecewise members are obtained. The theoretical and experimental results for FRP strengthening steel members are compared in Section 4.

## 2. Review of theoretical approaches to analysis of stability of compressed piecewise members

### 2.1. Elastic buckling

In elastic buckling analysis allowing for large deflections, the member's load-deflection curves exhibit a constant increase, gradually approaching the elastic buckling load without a peak because the initial geometrical imperfections and material nonlinearity are not considered. The elastic buckling load $P_e$ of piecewise members can be determined using either approximate or precise methods. Harvey [18] and Hoblit et al. [19] proposed methods of quickly computing the critical buckling load of members with a finite number of steps. Later, other approximate methods, including the energy method and numerical method, were developed [14–16]. In the energy method, the elastic strain energy is calculated from a presumed deformation curve of the piecewise member, and the governing equation is established based on the principle of minimum potential energy. The accuracy of the calculation relies on how closely the shape function coincides with the real deformation shape of the piecewise member. When the order of the shape function is increased, the accuracy increases, and the calculation process becomes more complicated. The static method is an accurate means of obtaining the elastic buckling load. Using this method, Xie [20] calculated the $P_e$ of piecewise members composed of a homogenous material with pinned-pinned ends and variable cross-section in the middle supported by an elastic spring. In this paper, the static method is used to derive an exact solution of the elastic buckling load of piecewise members under various ideal boundary conditions, therein considering the influence of the interface properties between the core and the exterior part.