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Flexural-torsional stability of thin-walled functionally graded open-section beams

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ABSTRACT

This paper aims to present the flexural, torsional and flexural-torsional buckling of axially loaded thin-walled functionally graded (FG) open-section beams with various types of material distributions. Properties of metal-ceramic materials are described by a monotonic function of volume fraction of particles that varying across blade thickness according to a power law. The problem is formulated by using a two-noded 14-degree-of-freedom beam element. Governing buckling equations has been developed. Warping of cross-section and all the structural coupling coming from anisotropy of material are taken into account in this study. The critical load is obtained for thin-walled FG mono-symmetric I- and channel-sections with arbitrary distributions of material. As a special case, a numerical comparison is carried out to show the validity of the proposed theory with available results in the literature. In addition, effects of gradual law, ceramic core and skin, span-to-height on the buckling parameters of an axially loaded thin-walled FG open-section beam are also investigated.

1. Introduction

Functionally graded materials (FGMs) as special composites that have been widely used in a variety of structures and in many fields. Most commonly, it is strengthened by metal or ceramic constituent. The superiority of FGMs is able to shield against high temperatures, also provide stronger tensile strength and reduce the stress concentrations or jumps, optimizing, thus, likelihood of material failure. Its applications excel in optimal load-carrying capacity and have therefore been investigated, also become increasingly attractive in many engineering fabrication areas, for example, aerospace, mechanical engineering, etc.

In the recent years, with the increase of using FGMs, several studies [1–7] predominantly addressing various aspects of static, vibration and buckling for a FG beam have been performed.

Mehri et al. [8] proposed a semi-analytical solution for vibration and buckling FG truncated conical shell based on harmonic differential quadrature method. A similar study for isotropic, laminated, FG materials which using Donnell's shell theory was also carried out by Demir et al. [9]. More recently, Mehralian et al. [10] investigated the size-dependent torsional buckling of FG cylindrical shell on the basis of modified couple stress theory with the geometrical nonlinearity. Chakraborty et al. [11] developed a new beam finite element for analysis of functionally graded materials. Akgöz et al. [12,13] introduced a new beam model and new shear correction factors for performing functionally graded microbeams. Lezgy-Nazargah et al. [14] analyzed thermomechanical aspects of FGMs. The static, free vibration and dynamic response of FG piezoelectric beam using three-nodded beam element are investigated. Li [15] took into account both rotary inertia and shear deformation in studying static and dynamic behaviors of FG rectangular beam. According to these papers, the authors indicated that FGMs were beneficial or detrimental dependent on a specific design. Furthermore, the grading could improve thermal properties and reduce stress concentration. In particular, they showed that FGM core may mitigate or even prevent impact damage in structures and further yield a significant weight reduction.

With regard to thin-walled structures, buckling analysis plays an important role in evaluation of their failures. In many years, the stability of thin-walled beam was well established by Vlasov [16] and later various thin-walled laminated composite beam theories were developed by many researchers [17–21] which allow a separate modeling of in-plane and through-the-thickness properties to tackle better the numerical complexity of multi-layer composites.

Concerned with isotropic materials, buckling by bending and torsion separately occurs under axial load if the cross-section has two axes of symmetry. For thin-walled laminated composite, however, they are no longer uncoupled even for doubly symmetric section, and flexural-torsional buckling should be considered. The subject of buckling and stability was also addressed in the recent papers [22–28].

For thin-walled FGMs, center of gravity and shear center are mobile

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points which move and directly depend on geometrical and material input parameters [29,30]. It therefore causes many influences over buckling characteristics. For instance, if the centroid and shear center coincide in cross-section leading to uncoupled buckling modes with respect to the principle axes. On the other hand, these positions are susceptible to the transformation of materials and geometries such as mono-symmetric cross-section, simultaneous coupled flexural-torsional buckling exhibits strong effects on behavior of beam. Thus, there remains a need that can guarantee a reliable the critical buckling load for various material distributions and properties throughout the structures.

More recently, Lanc et al. [31] performed the global buckling behavior of thin-walled FG sandwich box beams. The analysis was conducted using Euler-Bernoulli beam theory for bending and Vlasov theory for torsion with various boundary conditions. Oh et al. [32] obtained the solution for vibration and instability of circular cylindrical thin-wall FG beams related to thermoelastic modeling. Librescu et al. [33] also dealt with similar problem in which the effect of pretwist and nonuniformity of the beam cross-section, transverse shear and secondary warping were considered.

In the framework of thin-walled FG analysis [30,34], this paper aims to present a general analytical model in studying the global buckling of axially loaded thin-walled FG open-section beams and a finite element method has been developed as a tool that preserving the computational efficiency of the overall analysis. The model is based on Vlasov's thin-walled bar theory, and accounts for all structural coupling coming from the material anisotropy and warping of cross-sections. Governing buckling equations are derived by means of the principle of the stationary value of total potential energy. The results are obtained for mono-symmetric I- and channel-sections. To access the theory, three types of material distributions are considered. In order to verify the results of this study, several numerical examples are presented and compared with those obtained from others available in the literature. Moreover, the effects of gradual law, span-to-height, ceramic core and skin thickness ratios on the buckling parameters of thin-walled FG beams are also investigated.

The paper is organized as follows. The coordinate systems, assumptions and strains of thin-walled open-section are briefly outlined in Section 2. The variational formulations and governing buckling equations are given in Sections 3 and 4. These sections also provide the constitutive equations for a thin-walled FG beam and several types of material distributions. A finite element method using two-noded 14-degree-of-freedom element is developed in Section 5. Parametric studies and numerical examples for verification of mono-symmetric I- and channel-sections are worked and discussed in Section 6. Finally, the paper ends up with some concluding remarks in Section 7.

2. Displacement fields

In order to build a general model for thin-walled beam, three different coordinate systems are considered: an orthogonal Cartesian coordinate global system (x, y, z), an orthogonal local coordinate system (n, s, z) and a contour coordinate axis as shown in Fig. 1. The (x, y, z) and (n, s, z) coordinate systems are related through an angle of orientation θ. Toward its modeling, the following assumptions are adopted

(i) The strains are assumed to be small.
(ii) The beam is linearly elastic and prismatic.
(iii) The contour of a cross-section does not deform in its own plane.
(iv) The shear strain of the mid-surface ($\bar{\gamma}_{zs}$) and normal stress in the contour (σ_{ss}) are small and can be neglected.
(v) The Kirchhoff-Love assumption is valid for each segment of the cross-section.
(vi) Local buckling and distortional buckling are not considered.

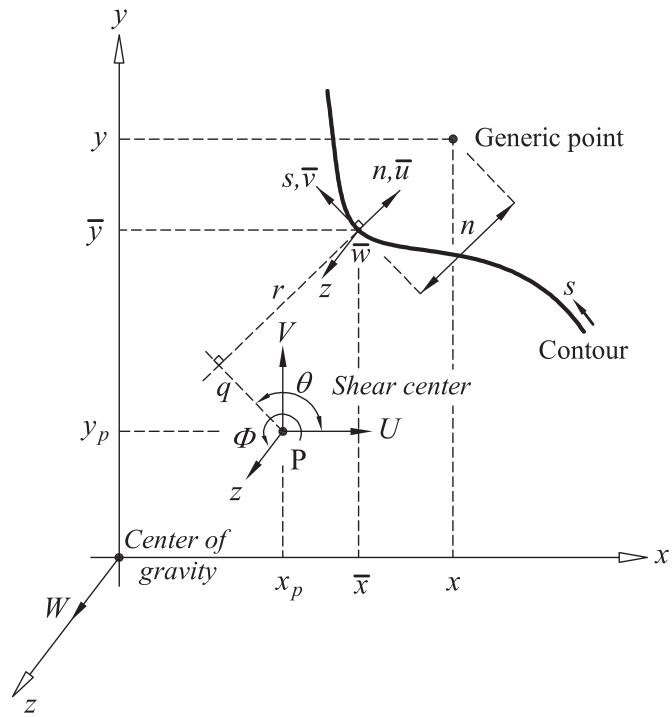

Fig. 1. Thin-walled coordinate systems.

Each segment of cross-section behaves as a thin shell in which the displacement of a generic point can be expressed in terms of location of the shear center by the following relations

$$x = x_p + (r + n)\sin\theta - q\cos\theta, \tag{1a}$$

$$y = y_p - (r + n)\cos\theta - q\sin\theta, \tag{1b}$$

where r and q are the distances from the pole P to that point normal to and parallel with the tangent to the contour, respectively.

The displacement components for any generic point on the profile section (u, v, w) can be calculated with respect to mid-surface displacements $(\bar{u}, \bar{v}, \bar{w})$ as

$$u(n, s, z) = \bar{u}(s, z), \tag{2a}$$

$$v(n, s, z) = \frac{\Gamma + n}{\Gamma}\bar{v}(s, z) - n\frac{\partial\bar{u}(s, z)}{\partial s}, \tag{2b}$$

$$w(n, s, z) = \bar{w}(s, z) - n\frac{\partial\bar{u}(s, z)}{\partial z}, \tag{2c}$$

where Γ denotes radius of curvature at the point consideration. If θ is constant along each element ($\Gamma = \infty$) the ratio $\frac{\Gamma + n}{\Gamma}$ equals to 1.

According to above assumptions (i)–(vi), the transverse displacements $\bar{u}$ and $\bar{v}$ of mid-surface in the contour coordinate can be calculated in terms of displacements U and V at the pole P in x- and y-directions, respectively, and the rotation angle Φ, as [18]

$$\bar{u}(s, z) = U(z)\sin\theta(s) - V(z)\cos\theta(s) - \Phi(z)q(s), \tag{3a}$$

$$\bar{v}(s, z) = U(z)\cos\theta(s) + V(z)\sin\theta(s) + \Phi(z)r(s). \tag{3b}$$

The out-of-plane shell displacement $\bar{w}$ can now be found from the assumption (iv). For each element of middle surface

$$\bar{\gamma}_{zs} = \frac{\partial\bar{v}}{\partial z} + \frac{\partial\bar{w}}{\partial s} = 0. \tag{4}$$

Substituting $\bar{v}$ in Eq. (3b) into Eq. (4) and integrating with respect to s from the origin to the arbitrary point on contour, the displacement $\bar{w}$ can be expressed as [18]

$$\bar{w}(s, z) = W(z) - U'(z)x(s) - V'(z)y(s) - \Phi'(z)\omega(s), \tag{5}$$