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Nonlinear buckling analysis of shallow arches with elastic horizontal supports

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ABSTRACT

The stiffness of the supports has an influence on the buckling modes and buckling loads of the arch. This paper investigates the in-plane nonlinear behavior and stability of shallow circular arches with elastic horizontal supports that are uniformly subjected to a radial load by the principle of virtual work. The three limiting shallowness values and the critical flexibility of the elastic horizontal supports are derived to differentiate the buckling mode. As the flexibilities of the elastic horizontal supports of an arch increase, the buckling load of the arch decreases, and the central radial displacement increases. And the buckling mode of the arch would then be changed from asymmetric bifurcation buckling to asymmetric bifurcation buckling after the occurrence of snap-through buckling. Then, snap-through buckling occurs, and finally, there is no buckling. Then the parameters including rise-to-span ratio(f/L) and load type are further investigated. An experimental model is designed to verify the analytical results. Finally, a design method for a shallow circular arch with elastic horizontal supports is proposed with the requirement of the elastic-plastic buckling load carrying capacity and the maximum horizontal displacement of the supports for engineering reference.

1. Introduction

The ends of the arch have a horizontal thrust when it bears a vertical load uniformly. Its mechanical behavior is different from that of the beam. The arches are dominated by compression rather than by flexure [1]. The bending moment uniformly generated by the vertical load can be converted to axial compression based on the form of the arch curve. Then, the bending moment will decrease, and the stress distribution of the arch section tends to be even. Therefore, arches can be considered thrust structures. The stiffness of the supports has an influence on the mechanical behavior of the buckling modes for the arch [2,3]. The arches are always simplified as can be either pin-ended arches or fixed arches. In a real project, An arch is often connected with other structural members such as roof sheeting systems, braces and purlins that provide restraining actions to the arch and significantly influence the buckling resistance of the arch [1,4,5]. When the lateral displacements and twist rotations of an arch are fully restrained, it may buckle in an in-plane buckling mode [1]. Research studies of the elastic buckling of arches have mainly been extended to arches elastically supported by other structural members. Jinnik AH and Haifang Xiang early proposed the linear buckling load of a circular arch with rotational end restraints [6,7]. An analytical study of the effect of support stiffening on column buckling loads was carried out by Raymond [8]. The rotational stiffness at the support was assumed to increase linearly with the compressive load. The snap-through instability of a shallow elastic arch was considered. Studies of the in-plane nonlinear elastic behavior and stability of elastically supported shallow circular arches that were subjected to a radial load uniformly distributed around the arch axis have been carried out by Bradford, Pi and Tin-Loi [9]. The criteria that distinguished between the buckling modes were obtained, the relationship between the limiting shallowness and the flexibility of the elastic supports was established, and the critical flexibility of the elastic radial supports was derived. Then, a comprehensive investigation of nonlinear buckling and postbuckling analyses of pin-ended shallow circular arches subjected to a uniform radial load or a central concentrated load with equal elastic rotational end-restraints and parabolic arches subjected to the uniform vertical load were presented [1,10–12]. An elastic-plastic finite element model was established to study the in-plane ultimate strength and stability of steel circular arches with elastic horizontal supports using large deformation theory by ANSYS. The initial geometric crookedness, residual stress and material inelasticity were considered in the investigation. The design equations proposed for pin-ended arches were used for

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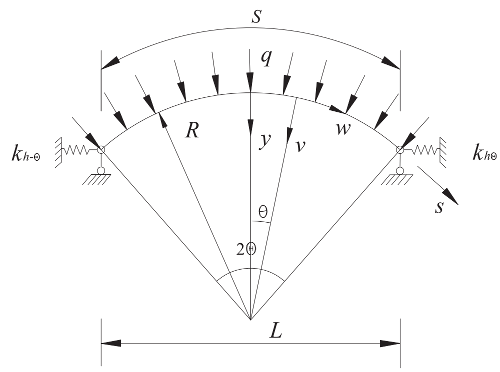

Fig. 1. Shallow circular arches with elastic horizontal supports.

Table 1
Buckling load and critical shallowness for different buckling mode.

Buckling mode	Buckling load	Critical shallowness
Symmetric buckling	$q = q_{\rm sn}(q_{\rm sn}R = \frac{\pi^2 EI_x}{S^2})$	$\lambda = \lambda_{sn} = \frac{\pi^3}{2\Theta\sqrt{16-\pi^4\Delta_1}}$
Asymmetric bifurcation buckling	$\bar{q} = \bar{q}_{sb1}$ $(A_1\bar{q}_{sb1}^2 + B_1\bar{q}_{sb1} + C_1 = 0)$	$\lambda_{\rm sn} < \lambda \le \lambda_{\rm sb1}$ $\lambda_{sb1} = \frac{4\pi^2}{\Theta}\sqrt{\frac{3A_1}{B_1^2 - 4A_1\Delta_2}}$
Snap-through buckling	$\bar{q} = \bar{q}_{sb2}$ $(A_2\bar{q}_{sb2}^2 + B_2\bar{q}_{sb2}^2 + C_2 = 0)$	$\lambda_{\rm sb1} < \lambda \le \lambda_{\rm sb2}$ $s.\ t.\ \mu\Theta = \pi;\ \bar{q}_{sb1} = \bar{q}_{sb2}$

elastically supported arches, and a simplified design criterion was presented [13,14]. The nonlinear behavior and in-plane stability of parabolic shallow arches with elastic rotational supports were investigated. A centrally concentrated load was applied to create compression in the supports. A nonlinear buckling analysis based on the virtual work formulation was carried out to obtain the critical load for both symmetric snap-through buckling and anti-symmetric bifurcation buckling [16].

The circular shallow arch and deep arch are distinguished by the rise-to-span ratio (*f/L*).When *f/L* does not exceed 0.2, the arch is a deep arch. The shallow arches experience significant geometric nonlinearities [17,18]. The deformations are significant prior to buckling, as noted by Pi and Trahair [19], so their effects on the in-plane buckling of shallow arches need to be accurately accounted for. These effects may significantly reduce the in-plane buckling resistance of shallow arches [2]. Non-linear stability behavior of functionally graded (FG) circular shallow arches subjected to a uniform radial pressure has been investigated by an analytical method [20]. Experimental studies on in-plane nonlinear behavior parabolic arches and circular arches were conducted to study the failure mechanism of arches and provide a basis for the design method by Guo [21,23]. However, The linear buckling analysis and numerical analysis were only used to investigate the in-plane ultimate strength of a shallow arch with elastic horizontal supports. The main purpose of this paper is to investigate the nonlinear buckling of shallow arches with elastic horizontal supports that are subjected to a radial load uniformly distributed around the arch axis by the principle of virtual work (Fig. 1). The dimensionless flexibility is proposed to study the effect of the elastic horizontal supports on the nonlinear elastic-plastic behavior and stability of the arch by numerical analysis. Then, a static test of a shallow arch with elastic horizontal supports is conducted to verify the analysis results. Finally, the design method of a shallow circular arch with elastic horizontal supports is proposed for engineering reference.

2. Nonlinear in-plane equilibrium equation

From the principle of virtual work the nonlinear in-plane equilibrium equations for a shallow arch with elastic horizontal supports at both ends and subjected to a radial load uniformly distributed around the arch axis can be obtained.

$$\delta V = \int_V E\varepsilon\delta\varepsilon dV - \int_{-\Theta}^{\Theta} qR^2\delta\tilde{v}d\theta + \sum_{i=\pm\Theta}\left(k_{hi}x_{hi}\delta x_{hi}\right) = 0 \tag{1}$$

where V is the volume occupied by the arch. E is Young's modulus of elasticity. R is the radius of the arch. q is a radial load uniformly distributed around the arch axis. Θ is half of the included angle of a circular arch. $k_{\rm hi}({\rm i}=\pm\Theta)$ is the stiffness of the elastic horizontal supports at both ends of the arch, $x_{\rm hi}({\rm i}=\pm\Theta)$is the horizontal displacement at both ends of the arch,$\delta(\ )$ denotes the Lagrange operator of simultaneous variations and ε is the longitudinal normal strain at an arbitrary point on a cross-section, which can be expressed as the sum of the membrane strain $\varepsilon_m = \tilde{w}' - \tilde{v} + \frac{1}{2}(\tilde{v}')^2$ and the bending strain $\varepsilon_b = -y\tilde{v}''/R$ [2], where $\tilde{w} = w/R$, $\tilde{v} = v/R$. w and v are the axial and radial displacements of the centroid of the cross-section respectively. $\theta$is angular coordinate. y is the coordinate of the point in the principal axes.

For the vertical deformation, the boundary conditions are

$$\tilde{v}\cos\Theta - \tilde{w}\sin\Theta = 0, \qquad \theta = -\Theta \tag{2}$$

$$\tilde{v}\cos\Theta + \tilde{w}\sin\Theta = 0, \qquad \theta = \Theta \tag{3}$$

For the horizontal deformation, the boundary conditions are

$$EA\varepsilon_m\cos\Theta - EA\varepsilon_m\tilde{v}'\sin\Theta - \left(\frac{EI_x}{R^2}\tilde{v}'''\right)\sin\Theta - k_{h-\Theta}(\tilde{v}\sin\Theta + \tilde{w}\cos\Theta)R = 0, \qquad \theta = -\Theta \tag{4}$$

$$EA\varepsilon_m\cos\Theta + EA\varepsilon_m\tilde{v}'\sin\Theta + \left(\frac{EI_x}{R^2}\tilde{v}'''\right)\sin\Theta + k_{h\Theta}(\tilde{w}\cos\Theta - \tilde{v}\sin\Theta)R = 0, \qquad \theta = \Theta \tag{5}$$

where A is the area of the cross-section.

For the radial direction, the bending moment is zero.

$$\tilde{v}'' = 0, \quad \theta = \pm\Theta \tag{6}$$

By substituting the boundary conditions given by Eqs. (2)–(6) into the differential equations of equilibrium derived from Eq. (1), the radial deformation can then be expressed as.

$$\begin{aligned}\tilde{v} = {} & \frac{\bar{q}}{\mu^2}\left\{\frac{\cos(\mu\theta) - \cos(\mu\Theta)}{\cos(\mu\Theta)} + \frac{1}{2}\left[(\mu\theta)^2 - (\mu\Theta)^2\right]\right\} \\ & + \frac{\bar{q}}{\mu^2}\left\{\begin{array}{l} -\dfrac{(\mu\Theta)^3(\sin\Theta)(\alpha_{h\Theta} - \alpha_{h-\Theta})\mu\theta}{(\mu\Theta)^2(\alpha_{h\Theta} + \alpha_{h-\Theta})\sin\Theta - 1} + \\ \dfrac{(\mu\Theta)^4(\sin\Theta)\left[4(\mu\Theta)^2\alpha_{h\Theta}\alpha_{h-\Theta}\sin\Theta - \alpha_{h\Theta} - \alpha_{h-\Theta}\right]}{(\mu\Theta)^2(\alpha_{h\Theta} + \alpha_{h-\Theta})\sin\Theta - 1} \end{array}\right\} \\ & + \frac{(\mu\Theta)^2(\alpha_{h\Theta} - \alpha_{h-\Theta})\cos\Theta}{(\mu\Theta)^2(\alpha_{h\Theta} + \alpha_{h-\Theta})\sin\Theta - 1}\theta \\ & - \frac{(\mu\Theta)^2\Theta(\cos\Theta)\left[4(\mu\Theta)^2\alpha_{h\Theta}\alpha_{h-\Theta}\sin\Theta - \alpha_{h\Theta} - \alpha_{h-\Theta}\right]}{(\mu\Theta)^2(\alpha_{h\Theta} + \alpha_{h-\Theta})\sin\Theta - 1}\end{aligned} \tag{7}$$

where