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Analytical evaluation of dynamic characteristics of unanchored circular ground-based steel tanks

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ABSTRACT

Natural and rocking frequencies of liquid-filled unanchored cylindrical steel tanks are determined using an analytical approach. In the proposed simplified mechanical model, the bottom plate of the tank is replaced by equivalent rotational and vertical springs. To evaluate natural frequencies, potential and kinetic energies of the liquid-filled tank are utilized and the eigen equation is solved by applying the Lagrange method and Ritz-type mode shapes for elastically supported cylindrical shells. Formulas are also derived to determine the rocking frequency based on a rotational rigid body motion and results are verified with available experimental data. Also, effect of the increase in kinetic energy of the tank during rocking motion on the natural frequencies of the tank is explored. To investigate the effects of tank geometrical aspect ratios on the structural responses, three models with height to diameter ratios of 0.4 (squat tank), 0.63 (medium tank) and 0.9 (slender tank) are considered. The results obtained using the proposed simplified mechanical model indicate that the method is capable of determining the rocking frequency of unanchored steel liquid-filled tanks with acceptable accuracy. Also, it is shown that at the onset of rocking mode, the fundamental natural frequency of the tank decreases considerably.

1. Introduction

Ground-based steel storage tanks are used extensively to store oil and chemical products. They may be anchored to their foundations or rest unanchored. Numerous research works have been carried out to determine the dynamic properties and seismic response of anchored tanks [1–4], however, similar studies on the unanchored tanks have been limited. The main parameter affecting the response of an unanchored tank during lateral seismic acceleration is the uplifting of the bottom plate and the induced rocking vibration mode.

Some experimental and theoretical work aimed at investigating the effect of uplifting of the bottom plate and rocking response of unanchored tanks have been reported in the literature. An early attempt was made by Shih [5] in which natural modes of anchored and unanchored tanks were evaluated both analytically and experimentally. The analytical natural frequencies of anchored tanks were determined by application of Hamilton method, Lagrange equations and two individual Ritz-type mode shapes. He also evaluated rocking frequency of an unanchored model under harmonic base excitation. Later, Manos and Clough [6] carried out a comprehensive experimental static tilting support and dynamic time history shaking table acceleration tests on large-scale tank models. Their study showed that coupling of the uplift mechanism which dominates the response of the unanchored tanks may result in very high compressive axial membrane stresses. Barton and Parker [7] applied F.E. numerical analysis to investigate dynamic response of unanchored tanks. Soil support under the plate was modeled by several low stiffness springs in tension. They reported that the uplift of bottom plate obtained in the F.E. numerical model was much smaller than the experimental data. Ishida and Kobayashi [8] also investigated rocking response of unanchored tanks using shaking table excitation on small-scale models. They reported that rocking of the unanchored tank models started in lower accelerations compared to those calculated by the rigid body motion theory. Later, Qu [9] reported on the results of shaking table experiments to determine the seismic response and uplifting of bottom plate in unanchored tanks.

On theoretical side, Natsiavas and Babcock [10] developed an analytical method to determine the tank fundamental frequency and hydrodynamic loads. They modeled uplifting of bottom plate using an equivalent rotational spring. Natsiavas [11] also derived a set of equations to describe the dynamic response of unanchored tanks using energy method. Lau and Zeng [12] and Lau et al. [13] applied Ritz-type

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shape functions in combination with F.E. method to analyze the bottom plate uplift. Later, Berhaman and Behnamfar [14] used F.E. numerical modeling to determine the failure modes of unanchored tanks. Their analyses were linear and to determine failure modes, maximum shell stresses and base plate rotation were compared with allowable values in design codes. Malhotra [15] and Ahari et al. [16] also presented an approximate method for analysis of unanchored tanks, modeling the bottom plate by a number of flexibly supported prismatic beams.

On the effects of uplift on fluid-structure dynamic interaction, Veletsos and Tang [17] evaluated additional hydrodynamic pressures induced by rocking of cylindrical tanks. In more recent years, Ozdemira et al. [18] applied nonlinear fluid-structure interaction formulations to determine seismic response of anchored and unanchored tanks. The effects of shell circumferential wave forms on the modal frequencies of tanks were also studied by Maheri and Severn [2] and Nachtigall et al. [19]. They concluded that the fundamental mode of the tank is associated with higher order circumferential wave forms and that current design codes could not be confidently applied to determine the fundamental frequency of the tank.

A review of the literature as outlined above reveals that save for the early work presented by Shih [5], no other work has concentrated on evaluation of natural frequencies and mode shapes of unanchored steel tanks. Also, rocking mode frequency of unanchored tanks, including partial uplifting of the bottom plate during rocking, has not been investigated. The main aim of the present study is to introduce a viable analytical method to evaluate natural and rocking frequencies of unanchored tanks. Also, the important effects of rocking mode on the natural frequencies of the fluid-filled tanks are investigated.

2. Proposed analytical method

An analytical approach was presented by Shih [5] for evaluating natural frequencies of liquid-filled unanchored tank, based on two individual mode shapes. In this section, a three mode approach is proposed to include the effects of support flexibility on mode shapes. Furthermore, equations are presented to determine frequencies of rocking mode and the effect of rocking on the natural frequencies of the tank.

2.1. Natural frequencies

2.1.1. Proposed three-mode eigen solution

The potential and kinetic energies of a liquid-filled steel tank are presented by Shih [5]. After determining the potential and kinetic energies, by using Hamilton principle and Lagrange equations, dynamic equation of motion may be obtained.

For anchored tanks, two independent mode shapes including lateral ($\varphi(x)$) and axial ($\psi(x)$) were considered by Shih [5]. He assumed the torsional mode to be exactly the same as the lateral mode. However, in this study, to consider the effect of flexibility of bottom plate of unanchored tanks on the mode shapes, it was necessary to define the torsional mode as a separate mode. Therefore, three independent; lateral ($\varphi(x)$), axial ($\psi(x)$) and torsional ($\phi(x)$) mode shapes are defined. Using Lagrange equations, the equation of motion in matrix form will be:

$$\begin{bmatrix} \widetilde{D}-\omega^2\widetilde{A} & \widetilde{E} & \widetilde{F} \\ \widetilde{E} & \widetilde{G}-\omega^2\widetilde{B} & \widetilde{H} \\ \widetilde{F} & \widetilde{H} & \widetilde{I}-\omega^2\widetilde{C} \end{bmatrix} \begin{Bmatrix} \widetilde{U} \\ \widetilde{V} \\ \widetilde{W} \end{Bmatrix} = 0 \tag{1}$$

In these equations, ω is the natural frequency of the tank. For two independent modes, formulation of matrix arrays in Eq. (1) are given in Appendix A [5]. By upgrading this matrix to include the proposed three modes, only $\widetilde{C}$, $\widetilde{H}$ and $\widetilde{I}$ arrays would change. These arrays are defined in Eqs. (2)–(4).

$$C_{ij} = \frac{\rho_s t_s R}{2} < \varphi_i \varphi_j > + \frac{C_{vn}\rho_l R^2}{2} < \varphi_i \varphi_j >_{H_L} \tag{2}$$

$$H_{ij} = \frac{E t_s R}{2(1-\nu^2)}\left\{ -\frac{n}{R^2} < \phi_i \phi_j > + \frac{t_s^2}{12R^2}\left[-\frac{n^2}{R^2} < \phi_i \phi_j > + \nu n < \varphi_i'' \phi_j' > \right] \right\} - 2(1-\nu) n < \phi_i' \phi_j' > \tag{3}$$

$$I_{ij} = \frac{E t_s R}{2(1-\nu^2)}\left\{ \frac{1}{R^2} < \varphi_i \varphi_j > + \frac{t_s^2}{12R^2}\left[R^2 < \varphi_i'' \varphi_j'' > + \frac{n^4}{R^2} < \varphi_i \varphi_j > - 2\nu n^2 < \varphi_i'' \varphi_j > + 2(1-\nu) n^2 < \varphi_i' \varphi_j' > \right] \right\} + \frac{\rho_l g}{2}\{ n^2 < \varphi_i \varphi_j >_{H_L} \} \tag{4}$$

In Eqs. (1)–(4), $\widetilde{U}$, $\widetilde{V}$ and $\widetilde{W}$ are displacement arrays in the lateral, torsional and axial directions, with $<A> = \int_0^L A dx$, $<A>_{H_L} = \int_0^{H_L} (H_L - x) A dx$. Also, R is radius, H_L is liquid level, H is height of the tank, t_s and t_b are thicknesses of wall and the bottom plate, ρ_s is tank shell density and n is the circumferential wave number. For simplicity, only the first mode in each direction is considered here. Therefore, order of the matrix in Eq. (1) would be three. The eigen solution of this matrix would produce three eigen values, smallest of which is regarded as the fundamental natural frequency of the tank.

2.1.2. Effect of support flexibility on mode shapes

In the following, evaluation of lateral and axial modes giving due consideration to the effect of support flexibility on the mode shape is discussed.

2.1.2.1. Lateral mode. For the lateral mode, in a simplified mechanical model, partial uplifting of the bottom plate is modeled with an equivalent rotational spring. Stiffness of this spring depends on the overturning moment of the tank against bottom plate uplift as shown in Fig. 1. In this simplified mechanical model, the tank wall is idealized as an equivalent flexural beam and the bottom plate is modeled with a rotational spring.

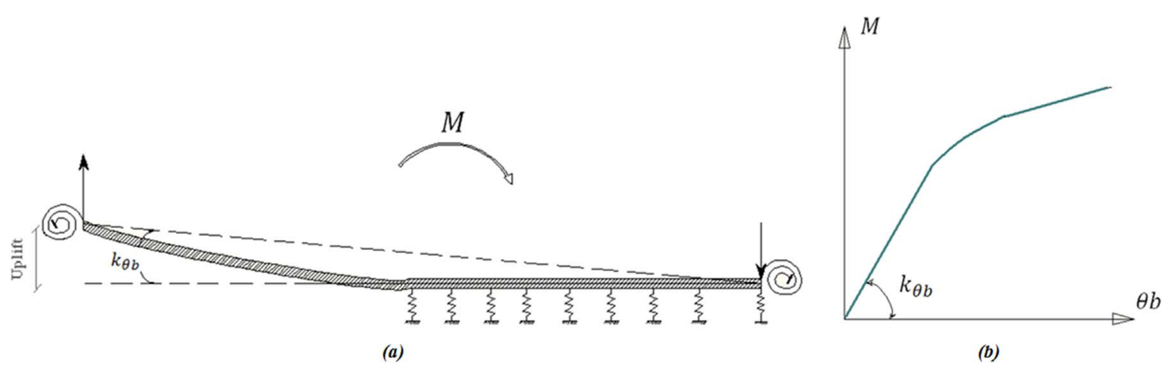

Fig. 1. (a) Uplifting of bottom plate and schematic representation of bottom plate rotation, (b) general form of the moment-rotation diagram of the bottom plate.