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Influence of the distortional-lateral buckling mode on the interactive buckling of short channels

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ABSTRACT

The present paper deals with an influence of the distortional-lateral buckling mode on the interactive buckling of thin-walled short channels with imperfections subjected to the bending moment when the shear lag phenomenon and distortional deformations are taken into account. A plate model (2D) is adopted for the channel section. The structure is assumed to be simply supported at the ends. A method of the modal solution to the coupled buckling problem within the Koiter's asymptotic theory, using the semi-analytical method (SAM) and the transition matrix method, was applied. The calculations and experimental preliminary tests were carried out for short channels.

1. Introduction

Beams are the fundamental element in steel structures that carry loads mainly via bending. In the majority of cases, a possibility to manage comparatively high loads with thin-walled beams is limited not by their strength only but mainly by stability.

The major development of research on stability of thin-walled isotropic structures took place in the 1970s and the 1980s. The exemplary papers dealing with local buckling of thin-walled structures are those written by Davids and Hancock [12]. Since the late 1980s, the Generalized Beam Theory (GBT) was developed extensively. At that time numerous studies employing such theories as: GBT, FSM (Finite Strip Method) and DSM (Direct Strength Method) [5,7,9,11,15,28,29,43] originated. In Refs. [2–4], a method for calculation of critical forces for pure bending of thin-walled beams, implemented on the basis of the Finite Strip Method (FSM), is presented, whereas decomposition of the buckling mode was conducted on the basis of assumptions taken from the GBT. This new method was referred to as the constrained Finite Strip Method (cFSM). The newest theoretical development trends in steel thin-walled structures are discussed in, e.g., [1,6,14,26,27,32–36].

Davies [13] described the development in research and analysis of thin-walled beams. The advancement in the buckling theory of thin-walled beams under buckling and compression was presented in Ref. [16]. In Ref. [10], models employed in the finite strip method are studied. The strength and local buckling of channel section and Z-section beams under bending were investigated numerically and experimentally in Ref. [46]. Stability of thin-walled beams and frames was investigated numerically and described analytically in Ref. [44].

Finite-element (FE) software packages have long been used to analyse thin-walled structures. FE models can simulate the actual structural behaviour closely and, therefore, replace experiments. FE models are used to produce data for evaluation and revision of current design formulae [30,31].

The results of experimental investigations of steel thin-walled beams are shown [40–42]. In Refs. [38,39], special attention was paid to the distortional-global interaction buckling.

Within the research project entitled"Experimental and numerical investigations of nonlinear stability of thin-walled composite structures" (DEC-2011/03/B/ST8/06447, founded by the National Centre for Sciences, Poland), experimental investigations were carried out for short composite channels subject to bending in the web plane (Fig. 1) as there is a lack of investigations devoted to the buckling analysis of such channel sections. The results pointed out especially to an effect of the global distortional-lateral buckling mode on postbuckling equilibrium paths. It was decided to explain that phenomenon in composite channels for a simpler case referring to steel channel sections under bending. In the present study, an influence of the distortional-lateral buckling mode on the interactive buckling of short steel channels is presented. The results of calculations are compared to the results of preliminary experimental tests..

The concept of interactive buckling (i.e., coupled buckling) that involves the general asymptotic theory of stability is fundamental for theoretical considerations. Among all versions of the general nonlinear theory, the Koiter's theory [17,18] of conservative systems is the most popular one, owing to its general character and development, even

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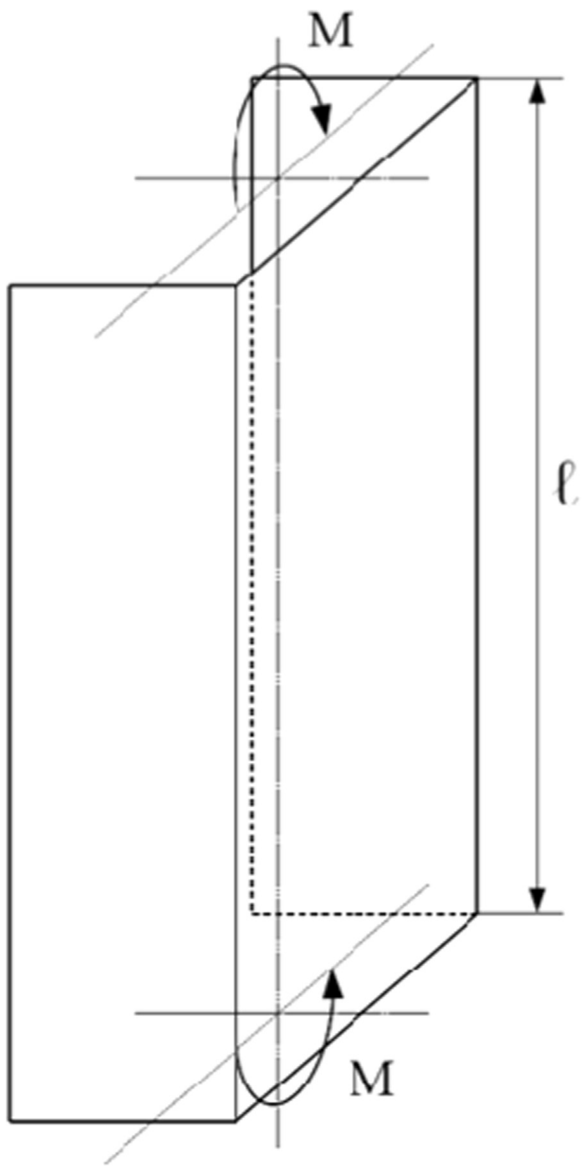

Fig. 1. Thin-walled channel subjected to bending in the web plane.

more so after Byskov and Hutchinson [8] formulated it in a convenient way.

The nonlinear stability of thin-walled channels in the first order approximation of Koiter's theory is solved with the modified analytical-numerical method (ANM) presented in Ref. [22]. The analytical-numerical method (ANM) should consider also the second order approximation of the theory in the analysis of postbuckling of elastic structures. The second order postbuckling coefficients were estimated with the semi-analytical method (SAM) [19] modified by the solution method in Ref. [24]. The attained results were compared to the results of the"complete" second-order analysis for isotropic structures [19,21,24].

In the present study, a plate model (2D) of the short channel is adopted to describe all buckling modes. Instead of the finite strip method, the exact transition matrix method and the numerical method of the transition matrix using Godunov's orthogonalization is used. The differential equilibrium equations were obtained from the principle of virtual works taking into account: Lagrange's description, full Green's strain tensor for thin-walled plates and the second Piola-Kirchhoff's stress tensor. The interaction between all the walls of structures being taken into account, the shear lag phenomenon and also the effect of cross-sectional distortions were included. The most important advantage of this method is such that a complete range of behaviour of thin-walled structures from all global to the local stability [20,21,23,37,45] can be described.

2. Formulation of the problem

A prismatic thin-walled isotropic channel built of plates connected along longitudinal edges and subjected to bending moment (Fig. 1) was considered. The girder was simply supported at its ends.

In order to account for all modes of global, local and coupled buckling, a plate model of thin-walled structures was assumed. It was assumed that the isotropic material (i.e., steel) the structure was made of obeyed Hooke's law.

For each plate component, precise geometrical relationships were assumed in order to enable the consideration of both out-of-plane and in-plane bending of the i-th plate [22–24]:

$$\begin{aligned} \varepsilon_{xi} &= u_{i,x} + \tfrac{1}{2}(w_{i,x}^2 + v_{i,x}^2 + u_{i,x}^2) \\ \varepsilon_{yi} &= v_{i,y} + \tfrac{1}{2}(w_{i,y}^2 + u_{i,y}^2 + v_{i,y}^2) \\ 2\varepsilon_{xyi} &= \gamma_{xyi} = u_{i,y} + v_{i,x} + w_{i,x}w_{i,y} + u_{i,x}u_{i,y} + v_{i,x}v_{i,y} \end{aligned} \tag{1}$$

and

$$\kappa_{xi} = -w_{i,xx}\kappa_{yi} = -w_{i,yy}\kappa_{xyi} = -2w_{i,xy} \tag{2}$$

where: u_i, v_i, w_i – components of the displacement vector of the i-th plate in the x_i, y_i, z_i axis direction, respectively, and the plane $x_i - y_i$ overlaps the central plane before its buckling.

The nonlinear problem of stability was solved with the asymptotic perturbation method. Let λ be a load factor. The displacement fields U and the sectional force fields N (Koiter's type expansion for the buckling problem [8,17,18]) were expanded into power series with respect to the dimensionless amplitude of the r-th mode deflection ζ_r (normalized in the given case by the condition of equality of the maximum deflection to the thickness of the first component plate h_1) (see [19–24,37,45]):

$$\begin{aligned} U &\equiv (u, v, w) = \lambda U_0 + \zeta_r U_r + \zeta_r^2 U_{rr} + \ldots \\ N &\equiv (N_x, N_y, N_{xy}) = \lambda N_0 + \zeta_r N_r + \zeta_r^2 N_{rr} + \ldots \end{aligned} \tag{3}$$

where the prebuckling (i.e., unbending) fields are U_0, N_0, the first nonlinear order fields are U_r, N_r (eigenvalues problems) and the second nonlinear order fields – U_{rr}, N_{rr}, respectively. The range of indices is $[1,J]$, where J is the number of interacting modes.

The boundary conditions referring to the simply supported beam-columns at their ends (i.e. $x = 0$; ℓ) are assumed to be:

$$\begin{aligned} &\int_0^b N_x(x=0, y)dy = \int_0^b N_x(x=\ell, y)dy = bN_x^{(0)} \\ &v(x=0, y) = v(x=\ell, y) = 0 \\ &w(x=0, y) = w(x=\ell, y) = 0 \\ &M_x(x=0, y) = M_x(x=\ell, y) = 0 \end{aligned} \tag{4}$$

The first condition in Eq. (4) means that the external loading is not subjected to any additional increment.

If the structure contains the geometric imperfections $\overline{U}$ (only the linear initial imperfections determined by the shape of the r-th buckling modes), where $\overline{U} = \zeta_r^* U_r$, then the total potential energy can be written in the form [8,19–24,37,45]:

$$\begin{aligned} \Pi = &-\frac{1}{2}M^2\overline{a}_0 + \frac{1}{2}\sum_{r=1}^{J}\overline{a}_r\zeta_r^2\left(1 - \frac{M}{M_r}\right) + \frac{1}{3}\sum_p^J\sum_q^J\sum_r^J \overline{a}_{pqr}\zeta_p\zeta_q\zeta_r \\ &+ \frac{1}{4}\sum_r^J \overline{b}_{rrrr}\zeta_r^3 - \sum_r^J \frac{M}{M_r}\overline{a}_r\zeta_r^*\zeta_r \end{aligned} \tag{5}$$

When the following notations are introduced:

$$a_{pqr} = \overline{a}_{pqr}/\overline{a}_r \; b_{rrrr} = \overline{b}_{rrrr}/\overline{a}_r \tag{6}$$